TULIPS, CONTINUED

1. Pricing tulips

- A tulip, planted, produces γ tulips next period.
- The demand curve for tulips of this variety by gardeners is $Q = a - bY$, where Q is the price per bulb and Y is the quantity sold.
- It costs c per bulb planted to plant bulbs and care for them until harvest.
- The discount rate is $1 + r$, and we use Q_t to designate the price of a bulb at time t.
- Problem: at time 0, there is only one bulb of this variety. Find its price.

2. Working it out

- We could always sell it at time 0 to a gardener, at price Q_0. Or, we could plant it, pay the cultivation cost c, and have γ bulbs, worth $Q_1\gamma$, at time 1. So a planted bulb's has a current price satisfies $Q_0 + c = \gamma Q_1/(1 + r)$. The demand curve implies we cannot sell any bulbs to gardeners at prices above a, so if $Q_0 > a$, all the bulbs will be planted by breeders.
- This reasoning will apply at every t. Since the demand curve implies that $Q_t \leq a$, so long as $Q_t > a$, no bulbs will be sold to gardeners. So long as any bulbs are being planted, the price of bulbs will have to satisfy $Q_t = (1 + r)(Q_{t-1} + c)/\gamma$
- So long as there are no sales to gardeners, the total available stock of bulbs is $Y_t = \gamma^t$.
- Assume $\gamma > (1 + r)$. Then the equation for Q implies that Q converges exponentially to its steady state value of $c(1 + r)/\gamma$. Once the price sinks below a, some of the stock is sold each period, so the exponential growth of Y ceases, and Y converges to its steady state value of $a - bc(1 + r)/\gamma$.

3. Determining Q_0

- What we've done so far tells us how to take any initial Q_0 and compute a time path for Q and Y. The Q path will always converge to the steady state value. The higher the initial Q, the longer Y grows exponentially.
- If Q_0 is too high, there is too large a stock of bulbs available when the price finally gets below a, and not enough are sold to prevent Y from continuing to grow indefinitely.
- If Q_0 is too low, Y is too small when Q falls below a, and there is not enough stock to satisfy demand.

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- There is only one initial value for Q that makes Y converge to a finite value.

4. Examples

```
tulips <- function(Q,nt) {
  ## demand:  P = a - b * Y
  a <- 10
  b <- .01
  c <- .5                                    # cost
  gam <- 3                                 # growth
  r <- .05                                   # interest rate
  q <- rep(0,nt)
  y <- rep(0,nt)
  y[1] <- 1
  q[1] <- Q
  for (it in 2:nt) {
    q[it] <- (q[it-1]+c)*(1+r)/gam
    if ( q[it] > a ){
      y[it] <- gam * y[it-1]
    } else {
      y[it] <- gam * (y[it-1] - ( a - q[it]) / b)
    }
  }
  return(ts(cbind(y,q)))
}
```

5. Numbers

- steady state Q is 2.1
- steady state y is 3423
- steady state sales to gardeners is 790

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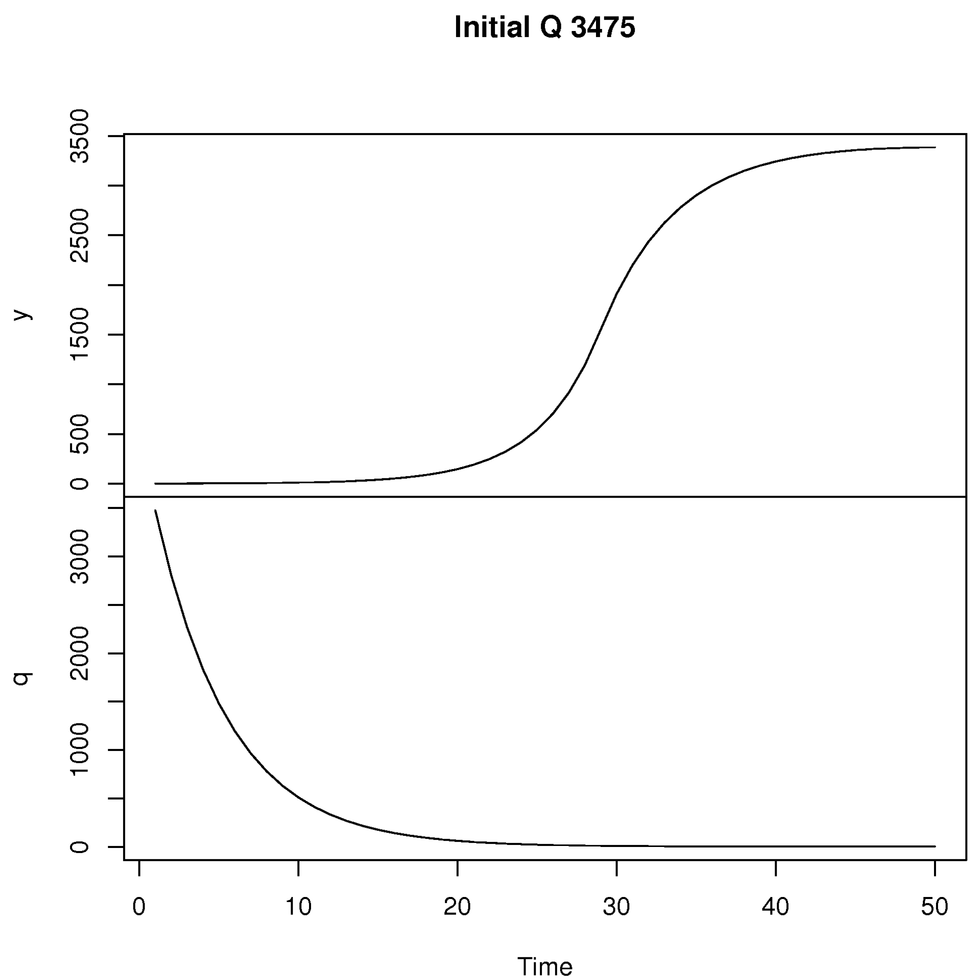
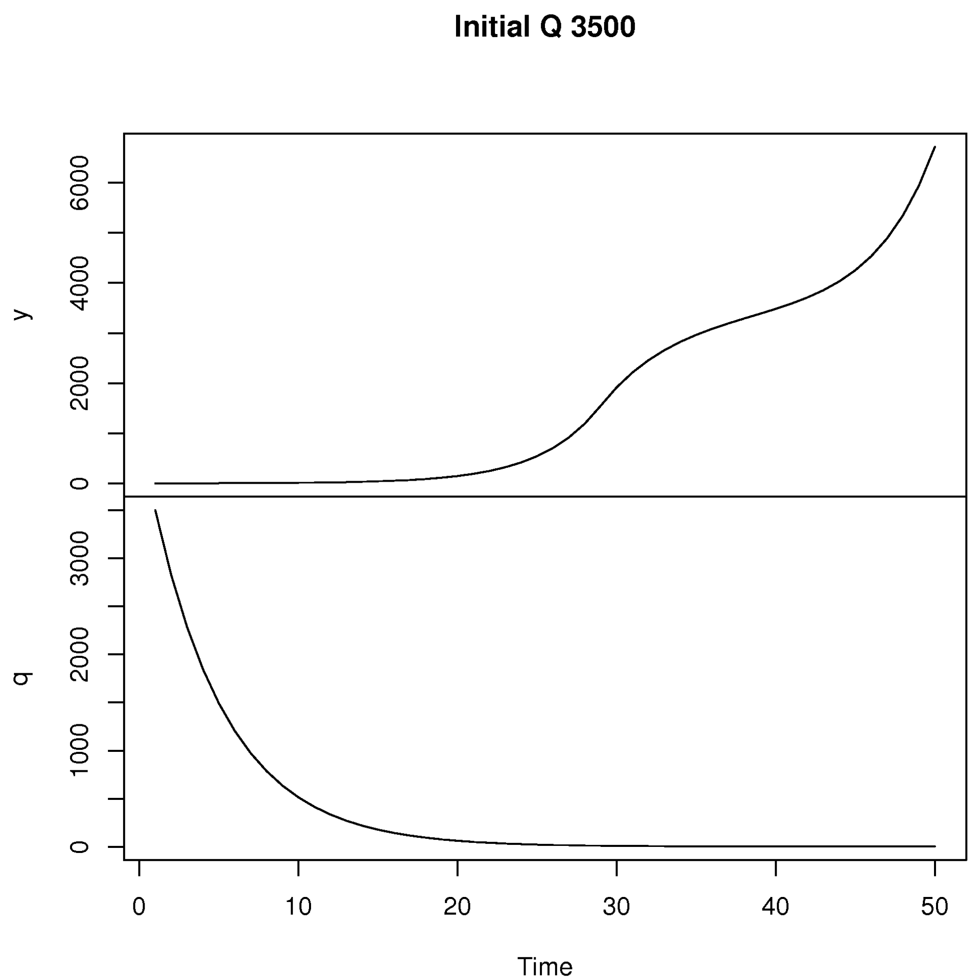
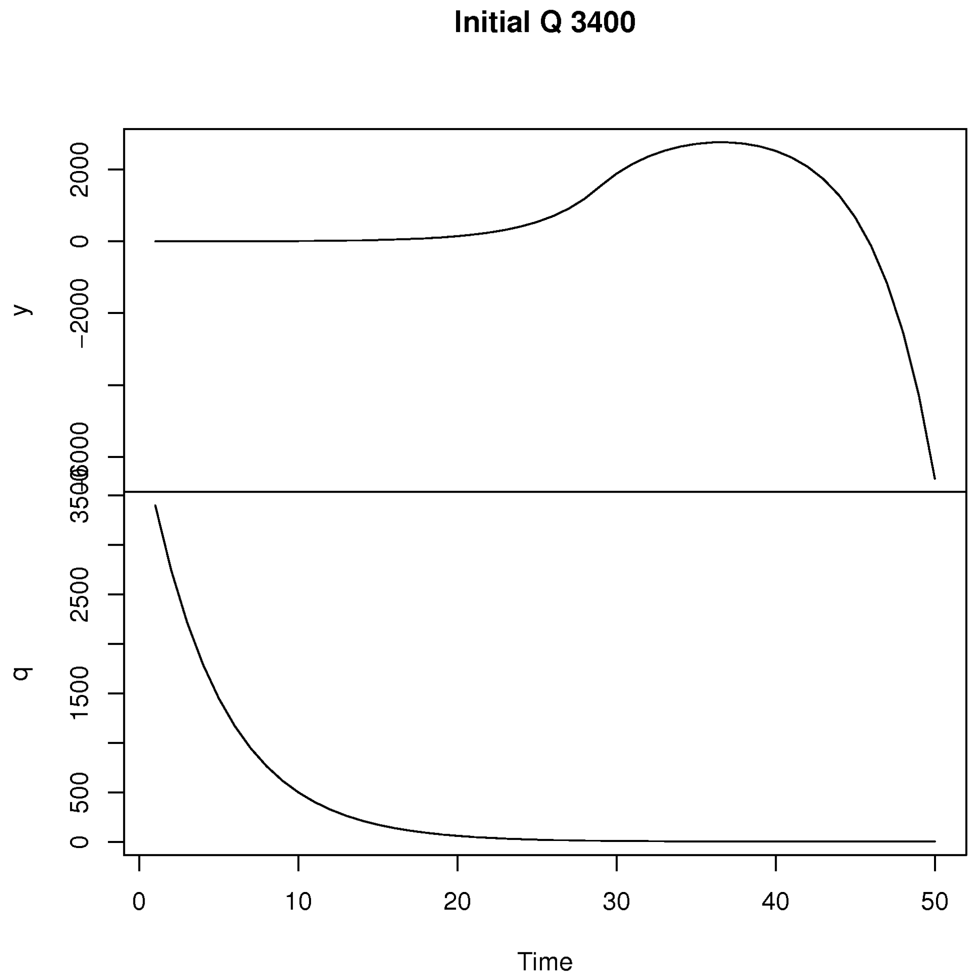

Initial Q 3475
y
0
500
1500
2500
3500
q
0
1000
2000
3000
0
10
20
30
40
50
Time

7


Initial Q 3500
y
0
2000
4000
6000
q
0
1000
2000
3000
0
10
20
30
40
50
Time

8


Initial Q 3400
y
2000
0
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q
3500
2500
1500
500
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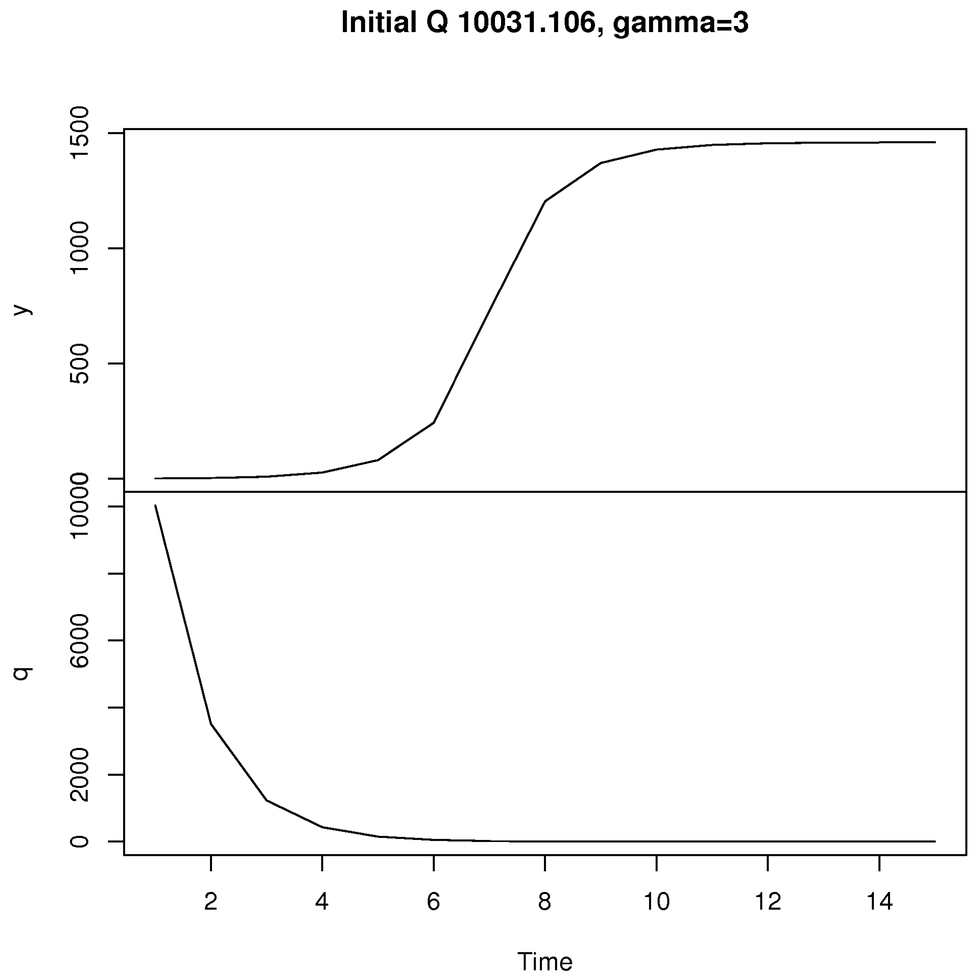
Time

9



10. How market participants must reason

- They have to guess conditions many seasons ahead, when there are finally enough bulbs to bring to market.
- Small changes in the demand curve, the interest rate r, or the reproduction rate γ have big effects on the current price.
- So it is easy to see why seemingly minor bits of information can have big effects on current prices.
- Market participants with modestly different views about demand or γ may have sharply different views about the appropriate current price, and thus be ready to enter deals that amount to "bets".
- Bets via lending, futures contracts.

11. VARIANTS TO THINK ABOUT

- How much difference would it make if the owner of the first bulb of this variety could patent it, thereby obtaining a monopoly on it?
- What happens if a bulb that was previously in steady state, with constant price and sales, but the interest rate increases?
- What if the interest rate increase is so big that now $(1 + r)/\gamma > 1$?