CASCADES AND BAYES' RULE

1. Bayes' Rule

Suppose we start out believing that θ has p.d.f. $p(\theta)$. We get a chance to observe the random variable Y, whose distribution depends on θ: Its conditional pdf is $q(y \mid \theta)$. The joint pdf of θ and Y is then, according to the rule that a joint distribution is the product of the marginal, or unconditional, pdf of one variable with the conditional pdf of the other, $q(y \mid \theta)p(\theta)$.

$$\text{Bayes' rule:} \qquad r(\theta \mid y) = \frac{p(\theta)q(y \mid \theta)}{\int p(\theta)q(y \mid \theta)\, d\theta}\,.$$

2. GPRM

t: indexes time, and people
S_t: signal observed by agent t, either 0 or 1
p_H: probability of $S_t = 1$ conditional on H being true state
p_L: probability of $S_t = 0$ conditional on L being true state
B_t: agent t action (either "buy", $B_t = 1$, or "sell", $B_t = 0$)

3. GPRM setup continued

- Agents see previous B_t's, not S_t's.
- Agents want to set $B_t = 1$ when H is the true state, $B_t = 0$ when L is the true state
- So agent t will set $B_t = 1$ when the conditional probability that the state is H, given the S_t and the history $\{B_s\}$ for $s = 1, \dots, t-1$, is above .5.
- GPRM assume that when the conditional probability is .5, the agent randomizes, choosing either value of B_t with equal probability. ("Trembling hand" reasoning suggests following one's own signal.)

4. Efficient use of information with publicly observed S_t

- We have observed a sequence $\bar{S}_t$ of S_t values, with n ones and $t-n$ zeros.
- $q(\bar{S}_t \mid H) = p_H^n(1-p_H)^{t-n}$
- $q(\bar{S}_t \mid L) = (1-p_L)^n p_L^{t-n}$
- If $p_H = p_L$, the odds ratio is

$$p_H^{2n-t}(1-p_H)^{t-2n} = \left(\frac{p_H}{1-p_H}\right)^{2n-t}\,.$$

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