Alternate Answer to Problem 3

You had the option, if you submitted problem 3 late, of proving that if X_t is a martingale,

$$\text{Cov}(X_t, X_s) = \sigma_{ts} = E[(X_t - \mu)(X_s - \mu)] = \sigma_{tt}\ , \text{ all } t \leq s\ . \tag{1}$$

Once you had this result, it was easy to see that only the covariance matrix in (b) met the condition.

The proof is as follows.

$$\begin{aligned}\sigma_{ts} = E[(X_t - \mu)(X_s - \mu)] &= E\big[E_t[(X_t - \mu)(X_s - \mu)]\big] \\ &= E\big[(X_t - \mu)E_t[X_s - \mu]\big] = E[(X_t - \mu)^2] = \sigma_{tt}\ ,\end{aligned} \tag{2}$$

where the second equality follows from the law of iterated expectations, the third from the fact that, conditional on information at t, $X_t - \mu$ is known, thus non-random, and can therefore by factored out of the argument of the E_t, and the fourth from the fact that X is a martingale.