

### COMMENT ON THE TAKEHOME FINAL EXERCISE

Everyone who did the takehome problem misunderstood the model, in the same way. The problem setup describes  $\Lambda_i$  as diagonal for each  $i$ , not as a scalar matrix for each  $i$ , but everyone programmed the model as if  $\Lambda_i = \lambda_i I$  for each  $i$ . This would not be so bad, except that the model is not identified with this restriction. This is a model with identification through heteroskedasticity, which we discussed in class, making the point that identification requires that every diagonal element of the  $\Lambda_i$  changes across  $i$ . The class notes did not emphasize that the *ratios* of changes in the diagonal elements have to differ across  $i$ , but that is clear from the argument for identification that was given in class.

Because of the lack of identification, MCMC will not converge, and the trace plots people displayed make it clear that convergence was not happening. Also, the lack of identification meant that permutation issues were a prominent problem in this mistaken version of the model, whereas they are not in the correct version of the model. (That is, in the 50,000 draws I did, a normalization rule making the first shock the one with the largest absolute size of effect on GDP never changed any draw. Since I did this check only after making draws, we know that the MCMC chain would eventually have sampled from the second peak with re-ordered shocks, but it is apparently well separated from the peak I found.)

And with this scalar- $\Lambda_i$  model, the impulse responses are *exactly* the same across countries, except for a single scale factor. The idea of the model is that the relative importance of the two types of shocks might differ across countries, and that this could deliver identification.

Though this is all discouraging, it would have been a truly inspired performance if anyone had got all this right, because of serious omissions on my part. I thought I was giving you work on a model that you should know or easily refresh yourself on. But though I had talked about it in class, with fairly complete notes, I realized as I graded that those notes never made it to the course web page. They are there now, but were not available during the exam.

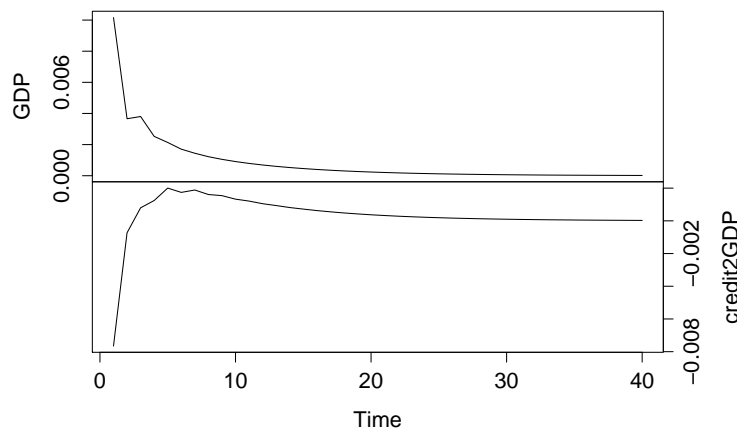
On top of that, I realized as I tried to do the work myself that the code I gave you had several annoying flaws. This code was a starting point for work last summer, but I had forgotten that in this form it was not tested and had some errors in it. Those taking the exam nonetheless seemed to get it to work. There were problems because the code did not deal correctly with the case where there is no constant term and no other  $x$  variable, which was the case in this exercise, and with where a zero got added at the beginning of `Tsigbrk`.

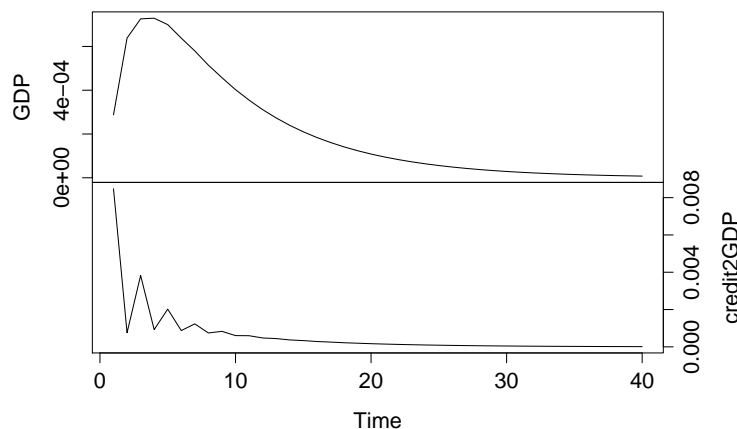
Impulse responses to the structural shock for the country for which we have normalized  $\lambda_{i,i}$  to zero are found, assuming `shvout` is the output from a run of `SVARhtskdmd`, by using

```
By <- tensor(solve(A0), shvout$var$By, 2, 1)
## or, to avoid the tensor package,
## By <- solve(A0, matrix(shvout$By, nrow=2))
## By <- array(By, c(2,2,2))!
smat <- solve(A0)!
resphat <- impulsdtrf(var=list(By), smat=smat)
```

A common mistake was to produce responses to reduced form shocks rather than structural shocks, by constructing an `smat` from the Cholesky decomposition of `solve(crossprod(A0))`. In this case, the first structural shock was pretty close to the first shock from the triangularized reduced form, but the second was different. Triangularization of the reduced form does produce orthogonal shocks in a standard VAR, but in the correct version of this model the triangularization is different in each country, while  $A_0$  transforms the raw reduced form residuals in the same way in each country. Of course in the scalar- $\Lambda_i$  version of the model, since  $A_0$  is not identified, the impulse responses to structural shocks probably would have looked strange, with very wide error bands.

**Modal Responses to first shock, US**



**Modal Responses to second shock, US**

The strongest shock, explaining most variance in both variables, is one that increases GDP and decreases credit-to-GDP. GDP returns monotonically to steady state, while credit-to-GDP rises slightly above steady state and then comes back down. Because the credit-to-GDP ratio declines less in log units than GDP rises, credit itself is initially expanding, just slower at first than GDP. The other shock, which is much smaller in all countries, moves both variables in the same direction. So there is little evidence of the kind of story the exercise put forward as an hypothesis. There are no positive credit shocks that predict future negative GDP growth, at any horizon.

Nonetheless, there are two distinct shocks that vary in size across countries. The modal  $\text{lmd}$  matrix that I got, normalized on the US as country 5, was

Spain	France	Italy	Japan	US	UK
2.08	2.11	2.59	1.80	1.00	0.91
0.44	0.13	-0.01	0.14	1.00	-1.27

Note that I normalized the two diagonal elements of  $\text{lmd}[\cdot, 5]$  to 1 instead of 0, forgetting that  $\text{lmd}$  gets exponentiated, and used a Cauchy prior on  $\text{lmd}$ , whereas I meant to normalize  $\exp(\text{lmd})[\cdot, 5]$  to 1 and to put the Cauchy prior on  $\exp(\text{lmd})$ . The prior does not have much influence on results, though.

This suggests that identification via heteroskedasticity is working well here, though it leaves open the question of how to interpret the shocks. Mine would be that bursts of GDP growth generally generate modest credit expansion that quickly catches up with the GDP growth, but that there is also noise in the credit variable that has little relation to GDP. This leaves no room for the idea that rapid credit expansion predicts future low GDP growth, at least in this simple linear model.

Calculation of autocovariance functions with `acf` shows little residual serial correlation, except that in several countries credit-to-GDP seems to have seasonality not picked up by the model. That is, they have fourth order (and in one case also 8th and 12th order) autocorrelation in the residuals. This may well be distorting results. A more elaborate model would be required to account for seasonality differing across countries.

Defects of the model for this application, besides the differencing of the data to get rid of constants and the lack of attention to seasonality, include

- Minnesota prior used as if we expect unit roots. Maybe inappropriate with differenced data.
- No recognition of the possibility that shocks are correlated across countries in the same time period.
- Error bands on the impulse responses are unrealistically tight, probably because restriction to a second order VAR is unrealistic.

Code I used is on the web site.