Take-Home Exam Answer

1. No unique answer to this essay question, of course. I looked for some specific and detailed use of the content of readings to support arguments you made, and for some critical thinking, either in the form of your own critique of what you had read or in the form of locating disagreement or conflicting implications across different readings. Most of the answers were somewhat disappointing in the degree to which they connected their claims to specific readings.

2. A truly complete answer to this problem required consideration of a maze or special cases. I did not expect exam answers to trace them all out. However an excellent answer should have located the critical values $\bar{b}$ and $\bar{\bar{b}}$ below and explained how they are related to the dynamics. It should also have recognized the possibility of multiple equilibria, with different *r* values, for some initial values of *b*. This is the economically interesting aspect of the example. It is a case where a stable price-pegging regime is possible, yet a high-interest rate regime, if it occurs, generates self-fulfilling expectations of devaluation.

You were meant to follow the specification of the last exercise, i.e. to assume a price-pegging regime to start with and to take *Y* and hence in equilibrium *C* as constant. Then we can conclude as in the last exercise that in equilibrium the nominal rate is just β plus the expected rate of devaluation, or

$$r = \beta + \psi \max\{0, \theta_0 + \theta_1(rb - \tau)\} \ . \qquad [1]$$

With *P* pegged at $\bar{P}$ before the policy switch, initial *b* is predetermined. We can use the policy equation (3) from the exam to replace τ by a function of *b*, giving us

$$r = \beta + \psi \max\{0, \theta_0 + \theta_1 \cdot (\phi_0 + (r - \phi_1)b)\} . \qquad [2]$$

One possible solution to [2] for given *b* is $r = \beta$. For this to work, we would clearly have to have the second term on the right of [2] 0 at $r = \beta$, i.e. have

$$\theta_0 + \theta_1 \cdot (\phi_0 + (\beta - \phi_1)b) \le 0 \ . \qquad [3]$$

Consider the leading case $\phi_1 > \beta$, which makes fiscal policy respond strongly enough to debt so that the price level could be sustained in the absence of any chance of devaluation of the debt. This requires large enough initial debt. We will label the critical value of debt at which [3] becomes an equality

$$\bar{b} = \frac{\theta_0 + \theta_1 \phi_0}{\theta_1(\phi_1 - \beta)} . \qquad [4]$$

So for $b \ge \bar{b}$, one solution is $r = \beta$.

We need to check next whether there might be a second $r \ge \beta$ that also solves [2] for the same *b*. This would occur if we could have

$$r=\beta+\psi\left[\theta_0+\theta_1\cdot\left(\phi_0+\left(r-\phi_1\right)b\right)\right]>\beta \qquad [5]$$

while still satisfying $b\geq\bar{b}$. Solving the equality in [5] for *r*, we can write inequality in [5] equivalently as

$$\frac{\beta+\psi\theta_0+\psi\theta_1\phi_0-\psi\theta_1\phi_1 b}{1-\psi\theta_1 b}>\beta\ . \qquad [6]$$

Now we need to introduce a second critical value of *b*, $\bar{\bar{b}}=1/\left(\psi\theta_1\right)$. If $b<\bar{\bar{b}}$, [6] reduces to $b<\bar{b}$. Thus in this case we conclude that there is a unique *r* corresponding to *b*, given by $r=\beta$ for $b\geq\bar{b}$ and by the left-hand side of [6], which exceeds *β*, for $b<\bar{b}$. This is a possibly unrealistic aspect of this model – it implies that large deficits appear only when debt is small, and that the deficits lead to fear of devaluation despite the small debt. Nonetheless this may capture some aspect of reality, since large deficits do make the country dependent on financial markets even if outstanding debt is small.

If $b>\bar{\bar{b}}$, [6] reduces instead to $b>\bar{b}$. So for $b>\bar{\bar{b}}$ and $b>\bar{b}$, we have two solutions for *r* for a given *b*, but when $\bar{\bar{b}}\leq b<\bar{b}$ we have no solution. This means that in these cases a jump to the new price level occurs instantly. When $\bar{b}\leq b\leq\bar{\bar{b}}$, we have a unique $r=\beta$ solution.

To understand the system's dynamics we plot $\dot{b}$ against *b*. When $r>\beta$, *b*'s dynamics are governed by

$$\dot{b}=\frac{\beta+\psi\theta_0+\psi\theta_1\phi_0-\psi\theta_1\phi_1 b}{1-\psi\theta_1 b}\cdot b+\phi_0-\phi_1 b\,, \qquad [7]$$

which we obtained by substituting the left-hand-side of [6] for *r* and the fiscal policy rule for *τ* in the government budget constraint (for constant *P*) $\dot{b}=rb-\tau$. This is not as complicated as it looks. It can be simplified to

$$\dot{b}=\frac{\phi_0+\left(\beta+\psi\theta_0-\phi_1\right)b}{1-b/\bar{\bar{b}}}\,. \qquad [8]$$

This expression leads us to define one more critical value of *b*, $\hat{b}=\phi_0/\left(\phi_1-\beta-\psi\theta_0\right)$. This could turn out to be negative if $\theta_0>0$. It has no necessary relationship to $\bar{b}$ or $\bar{\bar{b}}$. Equation [8] has a unique steady state at $b=\hat{b}$.

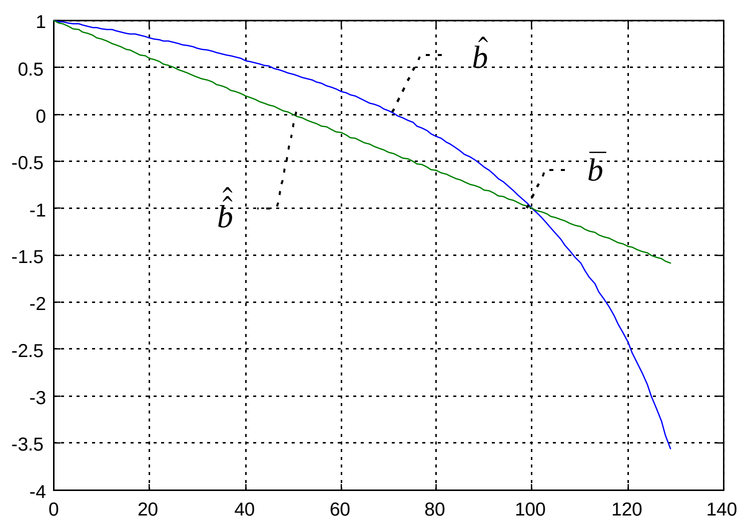
We also need to name $\hat{\hat{b}}=\phi_0/\left(\phi_1-\beta\right)$, the level of debt in a steady state with $r=\beta$ and zero probability of default.

There are some restrictions on the ordering of the four critical values of *b* in our $\phi_1>\beta$ case. When $\theta_0<0$, $\hat{\hat{b}}>\hat{b}$ and $\hat{\hat{b}}>\bar{b}$. And when $\theta_0>0$, $\bar{b}>\hat{\hat{b}}$ and either $\hat{\hat{b}}<\hat{b}$ or $\hat{b}<0$. This still leaves at least 8 possible orderings of the critical *b*'s (though it is conceivable that some of these could be eliminated also by further analysis). In determining how the system behaves the principles are: When $\hat{b} > \bar{\bar{b}}$, $\dot{b}$ is decreasing in b over $\left(0, \bar{\bar{b}}\right)$ when governed by the $r > \beta$ equation. Otherwise it is increasing on this range. As already noted, when $b < \bar{\bar{b}}$ and $b < \bar{b}$, the $r > \beta$ equation applies. When $\bar{\bar{b}} < b < \bar{b}$, there is no equilibrium value of r consistent with a finite δ, so the equilibrium must jump immediately to devaluation. When $\bar{b} < b < \bar{\bar{b}}$, there is a unique $r = \beta$ equilibrium, which may or not be stable, according to whether this interval contains $\hat{\hat{b}}$. When $b > \bar{b}$ and $b > \bar{\bar{b}}$, equilibrium is not unique. Either $r = \beta$ or $r > \beta$ equilibria are possible, and randomly timed jumps back and forth between them (sunspot equilibria) are also possible. When $\hat{b} < \bar{b}$ and $\hat{b} < \bar{\bar{b}}$, the $\hat{b}$ steady state is stable, but it entails $\delta > 0$ and therefore eventually gives way to devaluation. The only case that with certainty avoids devaluation is $\bar{b} < \hat{\hat{b}} < \bar{\bar{b}}$, with initial b in $\left(\bar{b}, \bar{\bar{b}}\right)$.

Plots of $\dot{b}$ against b for a case with $\phi_0 = 1$, $\phi_1 = \phi_1 = .07$, $\theta_0 = \theta_1 = .03$, $\beta = .05$, $\psi = .2$ appears below. In each figure the curved, blue line shows $r > \beta$ behavior, while the straight, green line shows $r = \beta$ behavior. In Figure 1, for $b < \bar{b}$, the curved, blue line is the only possible equilibrium. It has a stable equilibrium at $\hat{b} = 71$. For values of $b > \bar{b}$ shown in Figure 1, the straight green line takes over. However it also lies entirely below 0, so for initial b's in this range also there is a tendency to return toward the $\hat{b}$ steady state. In the neighborhood of this steady state, $r > \beta$, so $\delta > 0$, and thus devaluation occurs eventually, though not within a known finite time.

Figure 1

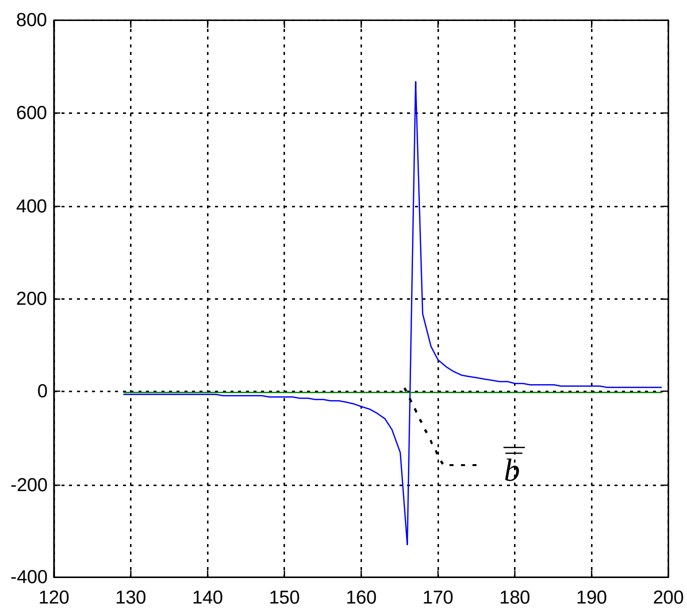
$\dot{b}$ as a function of *b*



In Figure 2 below we see the same plot as in Figure 1, extended out past $\bar{\bar{b}} = 167$. If initial b exceeds 167, both $r > \beta$ and $r = \beta$ equilibria are possible. (The green line for the latter is so close to zero on the scale of this graph that it is nearly invisible.) If the $r > \beta$ equilibrium occurs, $\dot{b}$ is positive, so b continues to rise further above $\bar{\bar{b}}$, δ remains positive, and eventually devaluation occurs. If the $r = \beta$ equilibrium occurs, $\dot{b}$ is instead negative, and the economy tends back toward the $\hat{b}$ steady state, again eventually producing a devaluation. Thus in this case no initial *b* is consistent with permanently avoiding devaluation.

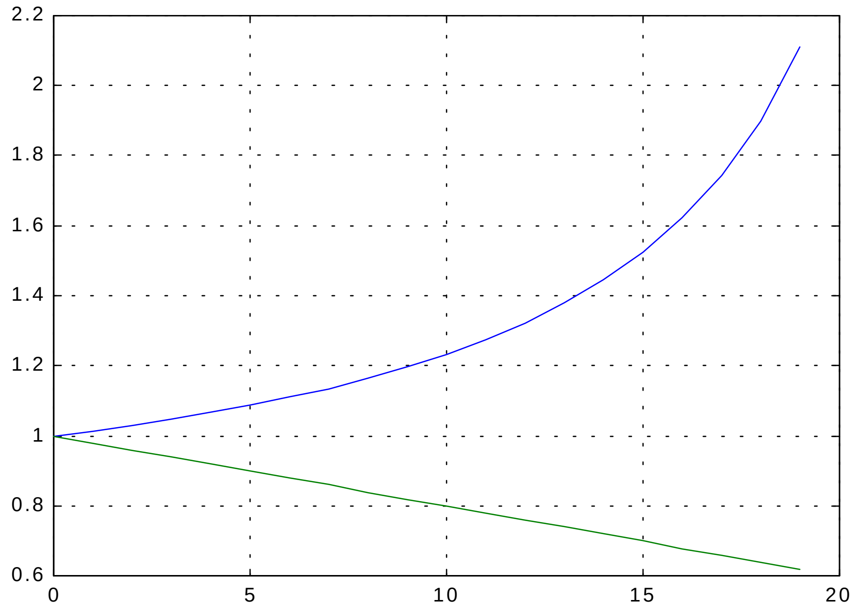
Figure 2

$\dot{b}$ as a function of *b*



A different configuration emerges when we set $\theta_1 = .2$, $\theta_0 = -.03$, leaving the other parameter values as above. Then $\bar{\bar{b}} = 25$, $\bar{b} = 42.5$, $\hat{b} = 38$, and $\hat{\hat{b}} = 50$. Plotting $\dot{b}$ for $b < 20$ gives us

Figure 3



None of the critical b values show on this graph. Because $b < \bar{b}$ and $b < \bar{\bar{b}}$ everywhere on this graph, the unique equilibrium corresponds to the upper, curved line. So b increases from any starting point in this range, with $\dot{b}$ in fact approaching plus infinity as b increases toward $\bar{\bar{b}}$. So from starting points in this range b increases and devaluation occurs within an a priori bounded time interval. If $b > \bar{\bar{b}}$ to start, Figure 4 below is relevant. In the range of initial b's between $\bar{\bar{b}} = 25$ on the left of this graph and $\bar{b} = 42.5$, there is no equilibrium with finite interest rate. The policy must switch immediately. For initial $b > \bar{b}$, both equilibria are possible. If the $\delta = 0$ equilibrium occurs, b converges to $\hat{\hat{b}}$ and stays there. Devaluation never occurs. However, it is also possible for the interest rate to emerge as higher than β, and in such an equilibrium b grows without bound and devaluation eventually occurs, though not within an a priori known time interval.

Figure 4