PHILLIPS CURVE EXERCISE

The exercise asks that you model the behavior if a policy maker who at each date estimates a Phillips curve by OLS and then choose a value of unemployment (or instead inflation) to maximize

$$-E\left[\sum_{s=0}^{\infty}\beta^t(u_t^2+\theta\pi^2)\right] \tag{1}$$

where u is unemployment and π is inflation. The policy maker solves the optimization as if his currently estimated Phillips curve coefficients are known with certainty and will prevail forever in the future. The specified form of the Phillips curve was (to match SWZ)

$$u_t = a_1 u_{t-1} + a_2 u_{t-2} + b_0\pi_t + b_1\pi_{t-1} + b_2\pi_{t-2} + \varepsilon_t\,. \tag{2}$$

You were asked to solve the problem once when the curve was estimated by OLS with u as dependent variable and once with the roles of π and u reversed. Then you were to compare the time path of u's or π's chosen by your optimizing policy maker with the data, The u and π values that result are not fed in to a model for the actual data. We just informally compare computed and actual data, experimenting with θ to get as good a match as possible. Though I didn't specify it, you should have just fixed β at some reasonable value for quarterly data — e.g. .99.

There was some ambiguity about timing. One choice, which makes the model somewhat simpler, is to suppose that at time t the policy maker sees ε_t. An alternative assumption about timing, perhaps more in keeping with the story that only past data are used to estimate (2), is that the policy variable value at t (u_t or π_t) must be chosen without knowledge of ε_t, which adds one more equation to the system.

The material below is left in because it does describe how to handle a model wheree commitment or time consistency is an issue. However, I posted it initially in response to a student's asking how to deal with lagged lambda's in the decision rule, which indicate a time consistency issue. This problem, though, has no time consistency issue, because there are no private agents forming expectations that could be violated. The correct solution to the policy-maker's problem therefore does not contain lagged Lagrange multipliers, and the material below in green is not relevant to this problem.

A final subtlety, noticed by at least one student, is that this is a model in which there is a time consistency issue. The policy maker with full ability to commit to future policy rules is not bound by the constraint (2) at dates before $t = 0$, and will therefore behave differently at time 0 from other dates. `gensys`, however, gives only the decision rule that prevails after these initial periods are over. This shows up in the `gensys` solution as lagged values of the Lagrange multiplier(s) in the decision rule.

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A natural way to handle this is to assume the initial date was long before the beginning of the data sample, and thus that non zero initial values of Lagrange multipliers are appropriate. Under this assumption, the lagged Lagrange multipliers can be found as functions of lagged u and π. Here's how,

If x_t us the full vector of variables in the model including Lagrange multipliers, the `gensys` solution will have the form

$$x_t = G_1 x_{t-1} + \xi_t \, . \tag{3}$$

$G1$ will be less than full rank, because eliminating unstable paths forces exact relations among some variables. If γ is an $n \times k$ matrix whose columns are eigenvectors of $G1$ corresponding to zero eigenvalues,

$$\gamma' x_t = \gamma' \xi_t . \tag{4}$$

This should supply enough equations to determine current Lagrange multiplier values from current u and π.