RATIONAL INATTENTION EXERCISE

(1) Suppose Y takes on the two values 0 and 1 with probabilities .25, .75, respectively. You must choose X. You lose $|X - Y|$ dollars if you choose X when Y is realized. X can take on only the values 0 or 1. You can give X and Y any joint distribution you like over these values, but mutual information between X and Y costs 50 cents per bit. Find the joint distribution of X and Y that minimizes expected total costs. Could you have done better if you were free to choose X values other than 0 and 1? Is there a critical finite cost of information above which no information would be collected? A non-zero threshold cost below which X is simply set equal to Y with probability one? [Note that there are only two degrees of freedom in this problem. It is probably best to solve it with a two-dimensional grid search.]

(2) X takes on the values $-1, 0, 2$ with the probabilities .2, .5, .3, respectively. Conditional on X, Y is distributed with pdf $e^{-(y-x)}$ on (X, ∞). Find the marginal pdf of Y and the conditional pdf of $X \mid Y$. (A pdf for X will be defined over the discrete measure on $(0, 1, 2)$, and thus just be the probabilities of those three points.) Find the mutual information between X and Y.

(3) You are on a quiz show. You are presented with four doors, behind one and only one of which there is a prize. You must pick one door. Then the show host opens two of the other doors to show that there is nothing behind them. He always does this, and always there is nothing behind the doors he opens, so you know now that the prize is either behind the door you originally selected or the other unopened door. How many bits of information (could be zero?) does the host give you by showing you the two doors with no prizes behind them? (And if allowed to switch to the other unopened door, should you?)