Debt, deficits and inflation in a low interest rate environment

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Zero real rates on government debt as a widow's cruse

- It seems that, when real rates on government debt are zero, debt finance is possible with no requirement for future primary surpluses to back the debt.
- Why, then, raise taxes when raising expenditures? Taxes distort, after all.
- This idea has recently emerged on the political left as "MMT", following a version of it articulated by VP Cheney: "Reagan taught us that deficits don't matter."

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- This idea has recently emerged on the political left as "MMT", following a version of it articulated by VP Cheney: "Reagan taught us that deficits don't matter."

- There might be some truth to it.

The "intemporal government budget constraint"

Step 1: Invoke the actual GBC ($\tau = x - g$):

$$\dot{B} = rB - \tau \ .$$

Step 4: Solve forward

$$B_t = \int_0^\infty e^{-rs} \tau_{t+s} \, ds \ .$$

Step 2: General solution

$$B_t = \int_0^\infty e^{-rs}\tau_{t+s}\,ds + \kappa e^{rt}$$

Step 3: Invoke "transversality"

$$e^{-rt}B_t \xrightarrow[t\to\infty]{} 0$$

There is no intemporal government budget constraint

- The equation usually given this label arises in an equilibrium model from combining the actual budget constraint with a necessary condition for private sector optimization, the actual private transversality condition.

- In some simple models this does turn out to be what I called above "transversality".

What if $r < \beta$?

- The private discount rate β, not r, appears in the private transversality condition. (Of course, in a richer model there is a stochastic private discount factor, and r also may be stochastically time varying.)

- The private TVC is, in a model with real capital whose equilibrium return matches β,
$$e^{-\beta t}(k_t + b_t) \to 0$$

- If we know $k_t \geq 0$ and $b_t \geq 0$, this implies $e^{-\beta t} b_t \to 0$.

Integrating forward with $r < \beta$

- We're not going to be able to invoke $e^{-rt}b_t \to 0$. Growth of (per capita) debt at a rate $r < \beta$ is not inconsistent with private sector optimization. So we have to rewrite the GBC with βb replacing rb on the right-hand-side. We also distinguish real debt b from nominal debt B and use the notation $\pi = \dot{P}/P$.

$$\frac{\dot{B}}{P} - b\pi = \dot{b} = \beta b - \tau - (\beta - r + \pi)b\,.$$

- We'll call $(\beta - r + \pi)b$ "seigniorage", σ.

A valid "intertemporal budget constraint"

$$b_t = \int_0^\infty e^{-\beta s}(\tau_{t+s} + \sigma_{t+s})\, ds$$

$$b_t + \int_0^\infty e^{-\beta t} g_{t+s} = \int_0^\infty e^{-\beta t}(x_{t+s} + \sigma_t) ds\ .$$

What is the right balance between x and σ?

- Friedman rule: Drive σ_t to zero, since government paper costs almost nothing, yet provides transactions services. Chari and Kehoe (1999) argue that the Friedman rule is optimal in a very general class of models, even when all available taxes are distortionary.

- In recent policy discussions about the size of the Fed balance sheet, some (e.g. Jeremy Stein) have argued that the balance sheet should be expanded until the rate on Fed reserve deposits matches the long rate, so the yield curve is flat.

- But if we take g and initial b as given, lowering σ requires raising x, and if x are distorting taxes, it is not obvious that this is a good idea.

An example model to look at the tradeoff

- Government debt in the budget constraint, as providing transaction services, so that it is return-dominated.
- Only input is labor, L.
- Just one kind of government liability: nominal, duration zero.

Private sector

$$\max_{C,B,L} \int_0^\infty e^{-\beta t}(\log C_t - L_t)\,dt$$

subject to

$$C \cdot (1 + \gamma v) + \frac{\dot{B}}{P} = (1 - \tau)L + \frac{rB}{P}$$

$$v = \frac{PC}{B}\,.$$

Private FOC's

$$\partial C: \qquad \frac{1}{C} = \lambda \cdot (1 + 2\gamma v)$$

$$\partial L: \qquad -1 = -\lambda \cdot (1 - \tau)$$

$$\partial B: \qquad \frac{-\dot{\lambda} + \lambda\beta + \lambda\dot{P}/P}{P} = \frac{\lambda\gamma v^2}{P} + \frac{r\lambda}{P}$$

Solving to eliminate λ,

$$C \cdot (1 + 2\gamma v) = 1 - \tau$$

$$\frac{\dot{C}}{C} + \frac{2\gamma\dot{v}}{1 + 2\gamma v} = \gamma v^2 - \beta - \frac{\dot{P}}{P} + r$$

($\gamma v^2 = R - r,$ where R is nominal rate on non-government debt)

Government

$$\max_{C,M,L,P,\tau} \int_0^\infty e^{-\beta t}(\log C_t - L_t)\, dt$$

subject to

private FOC's

$$SRC: \; C \cdot (1+\gamma v) + G = L$$

$$GBC: \; \frac{\dot{B}}{P} + \tau L = G + \frac{rB}{P}\,.$$

The Friedman rule is costly

Suppose there is an equilibrium with v, C and $\dot{P}/P$ all constant. Private sector behavior then requires

$$\gamma v^2 + r - \frac{\dot{P}}{P} = \beta \; .$$

Using the notation $b = B/P$, we can write the government budget constraint as

$$\frac{\dot{B}}{B} b + \tau L = (\gamma v^2 + r - \beta) b + \tau L = rb + G \; .$$

Can we get v down to zero?

C and L are bounded above for $\tau \geq 0$. (See private FOC). So revenue τL is bounded above.

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$v = C/b$ can approach zero only if C is near zero, which can't be optimal, or b grows arbitrarily large. But as steady-state v approaches zero, the government budget constraint approaches the form

$$-\beta b = g - \tau L \,.$$

Since τL is bounded, large enough b makes it impossible to satisfy the steady state budget constraint.

Inflating away initial debt

- Suppose that at time zero there is both liquidity-providing debt in the nominal amount B and another type of government debt H that provides no liquidity services, and hence pays a higher interest rate.

- At $t = 0$, in this flex-price, perfect-foresight model, the price level is determined by the government's announced future paths of tax rates and nominal interest rates (which determine real money balances and real debt) and the outstanding stock of nominal government liabilities, $B + H$.

Inflating away initial debt

- The government will need enough labor tax revenue to cover government spending G and any B-shrinkage or interest on B it decides to undertake.

- Optimally, it chooses *only* this much taxation, with nothing left over for H debt service, forcing the real value of H to zero.

- If initially a $H > 0$ was inherited from the past, agents will trade in their H holdings for B at $t = 0$, and P will generally jump at $t = 0$.

Other assumptions about $t = 0$?

- The behavior described on the last slide assumes the government can ignore public expectations in setting P_0. It does, however, preserve the nominal government budget constraint at $t = 0$, meaning that $\dot{B}_0 + \dot{H}_0$ is well defined and $B_t + H_t$ is continuoius from both left and right at $t = 0$.

- If the government can violate the flow budget constraint at $t = 0$ by repudiating existing debt and money, issuing a new debt and money with a senior claim on future revenue, it can achieve a better outcome.

- However, it would have to be believed when promising never to do this again. That is, such an initial default is a time-inconsistent policy, while initial inflation or deflation to achieve $H = 0$ while $B > 0$ is time consistent.

How do CK avoid this conclusion?

Their money-in-the-utility-function model has consumption and real balances entering utility separably as a homothetic function of C and m, and they emphasize the dependence of their result on this assumption. Our example model can be cast into the form they assume:

One sets $C^* = C(1 + \gamma v)$, solves for C as a function of C^* and m, and makes that an argument of the utility function.

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One sets $C^* = C(1 + \gamma v)$, solves for C as a function of C^* and m, and makes that an argument of the utility function.

The key assumption that allows their conclusion is that at $t = 0$, the government has zero net worth, and injects money into the economy by a one-time purchase of private sector, non-money liabilities. Their government therefore has assets exactly balancing its liabililties in the form of M. The required $\dot{M}/M = -\beta$ rate of contraction can be financed without taxation by steadily selling the government assets back to the private sector.

Example numerical solutions

γ	C	v	L	$\dot{P}/P$	U	τ	σ	$\gamma v/(1+\gamma v)$
0.001	0.69	2.98	0.993	-0.0111	-1.363	0.305	-0.0026	0.003
0.01	0.67	0.98	0.980	-0.0105	-1.375	0.314	-0.0072	0.010
0.1	0.62	0.34	0.944	-0.0086	-1.417	0.335	-0.0160	0.033
1	0.51	0.13	0.874	-0.0040	-1.548	0.361	-0.0159	0.112

Table 1: Optimal steady state with $G = .3$, $\beta = .02$

γ: transactions cost parameter; C: consumption; v: velocity PC/M; L: labor; $\dot{P}/P$: inflation rate; U: utility; τ: labor tax rate; σ: seigniorage revenue; $\gamma v/(1+\gamma v)$: proportion of consumption expenditure absorbed by transaction costs; G non-productive government expenditure; β: discount rate.

Example numerical solutions

γ	C	v	L	$\dot{P}/P$	U	τ	σ	$\gamma v/(1+\gamma v)$
0.001	0.97	0.74	0.973	-0.0194	-1.001	0.026	-0.0254	0.001
0.01	0.94	0.35	0.943	-0.0188	-1.005	0.054	-0.0507	0.003
0.1	0.87	0.16	0.882	-0.0173	-1.024	0.103	-0.0912	0.016
1	0.72	0.08	0.778	-0.0139	-1.104	0.166	-0.1293	0.072

Table 2: Optimal steady state with $G = 0$, $\beta = .02$

See notes to Table 1

Example numerical solutions

γ	C	v	L	$\dot{P}/P$	U	τ	σ	$\gamma v/(1+\gamma v)$
0.001	0.20	9.07	0.998	0.0623	-2.627	0.800	0.0013	0.009
0.01	0.19	2.92	0.994	0.0655	-2.665	0.801	0.0042	0.028
0.1	0.17	0.98	0.983	0.0755	-2.775	0.801	0.0129	0.089
1	0.12	0.34	0.965	0.0937	-3.060	0.794	0.0342	0.252

Table 3: Optimal steady state with $G = .8$, $\beta = .02$

See notes to Table 1

Technological progress

- If this is represented by exponential growth at the rate ν in the effectiveness of labor L, and if G itself grows at the same rate, the model does not change much.

- It increases the equilibrium real rate from β to $\beta + \nu$, and thereby increases the fiscal effort required to implement the Friedman rule.

- Because it results in steady growth in m, it reduces the "fiscal cost" of debt.

What if $t > 0$ and $H > 0$?

- If, in addition to interest bearing money, there is interest-bearing debt that provides no liquidity services after $t = 0$, The government can't jump to the $H = 0$ steady state. It has committed to a continuous path of prices.

- This puts us in the framework of Woodford's "timeless perspective", where private sector forward-looking FOC's have to be treated as holding continuously.

- Examining these dynamics should be feasible, and might be interesting. Does the model converge to the $H = 0$ steady state? If so, how fast?

The present situation

- To explain within this model the rise in real debt, accompanied by low inflation and low interest rates, that we have recently seen in many countries, we need to invoke a rise in demand for debt's liquidity services.

- This is plausible, given the 2008-9 experience and increased regulatory attention to balance sheets.

- But then it is not clear that the rise in debt is something to worry about — it could be optimal expansion of liquidity services.

So is deficit finance costly?

- What the model shows is that if the liquidity premium on government debt is endogenous, declining with expansions of the level of real debt, there is a cost to expanding the real debt, even when no taxes are "required" to back it.

- The rise in interest expense on the outstanding stock of debt when the level of debt rises has to be funded by increased taxes or reduced expenditure; otherwise inflation will undo the rise in the level of real debt.

- This can be true, even though "fiscal cost" is zero or negative. Seigniorage from the debt may be positive before the debt increase and positive afterwards, but without inflation, it declines, and the decline has to be made up in other parts of the budget.

The level of the debt vs sustained deficits

- In the numerical examples we looked at, while the Friedman rule is not optimal, the ratio of debt to output is high and optimal steady state seigniorage is zero or negative.

- But that high levels of debt may be optimal does not imply that running deficits is optimal.

- Increasing the level of debt without deflation requires running deficits for a finite time, after which primary surpluses are *higher* in the new steady state.

- Increasing levels of nominal debt without correesponding increases in taxation are possible only with increased seigniorage finance, which implies increased inflation.

What we've left out

- There are many other aspects of the question of whether low real rates on government debt suggest that we should not be concerned about expanding it.

- There is an intergenerational transfer aspect to government debt, which we have not addressed here.

- We have taken a very "black box" approach to modeling liquidity services of government debt. We don't understand very well why the demand for liquidity rose so much in 2008-9, and we have even less basis for confidence that the increased demand won't drop as quickly as it rose.

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References

CHARI, V., AND P. J. KEHOE (1999): "Optimal Fiscal and Monetary Policy," resreport 6891, National Bureau of Economic Research, http://www.nber.org/papers/w6891.