BUBBLES AND FUNDAMENTALS

1. DISCOUNTING RETURNS

- A promise to deliver money “later” is generally worth less than delivery of money now, even if the promise is completely credible.
- Such promises can be added and subtracted, so a promise to deliver $100 in two weeks and a promise to deliver $200 in two weeks should have prices that add up to the same as that of a promise to deliver $300 in two weeks.
- Furthermore, a promise to deliver, one week from now, exactly enough money to buy then a promise to deliver $100 one week further on, should have a current price exactly equal to the price of $100 delivered two weeks from now.

2. THE MARKET DISCOUNT FACTOR

- It turns out that these conditions, plus the assumption that any promise of this kind can be bought and sold freely, are enough to deliver the conclusion that there is a “market discount factor” $R_t$. This discount factor has the properties 1) that $R_t$ can be known at time $t$ and 2) that a promise to deliver the random return $y_{t+1}$ at time $t+1$ has the value at time $t$

$$
P_t = E_t \left[ \frac{y_{t+1} R_{t+1}}{R_t} \right].
$$

- If there is no uncertainty, or if participants in the asset market have no risk aversion, $R_t$ will be non-random. As an extreme simplification, we sometimes assume a constant interest rate, so that $R_t = (1 + r)^{-t}$ and the pricing relation becomes simply $y_{t+1} = (1 + r) P_t$.

3. If there is uncertainty, but the return $y_{t+1}$ is not uncertain (so we are talking about something like a treasury bill), then the return is the interest rate, and we have

$$
y_{t+1} = (1 + r) P_t = (1 + r) y_{t+1} E_t[R_{t+1}] / R_t \quad \text{i.e.}
$$

$$
1 / (1 + r) = E_t[R_{t+1} / R_t]
$$
4. Pricing a random-yield asset

- This asset pays the random yield $y_t$ each period.
- It is a stylized “stock” or “Lucas tree”.
- Its price $Q_t$ must satisfy

$$Q_t = E_t \left[ \frac{R_{t+1}(y_{t+1} + Q_{t+1})}{R_t} \right]$$