TULIPS, CONTINUED

1. PRICING TULIPS

- A tulip, planted, produces $\gamma$ tulips next period.
- The demand curve for tulips of this variety by gardeners is $Q = a - bY$, where $Q$ is the price per bulb and $Y$ is the quantity sold.
- It costs $c$ per bulb planted to plant bulbs and care for them until harvest.
- The discount rate is $1 + r$, and we use $Q_t$ to designate the price of a bulb at time $t$.
- Problem: at time 0, there is only one bulb of this variety. Find its price.

2. WORKING IT OUT

- We could always sell it at time 0 to a gardener, at price $Q_0$. Or, we could plant it, pay the cultivation cost $c$, and have $\gamma$ bulbs, worth $\gamma Q_1$, at time 1. So a planted bulb’s has a current price satisfies $Q_0 + c = \gamma Q_1 / (1 + r)$. The demand curve implies we cannot sell any bulbs to gardeners at prices above $a$, so if $Q_0 > a$, all the bulbs will be planted by breeders.
- This reasoning will apply at every $t$. Since the demand curve implies that $Q_t \leq a$, so long as $Q_t > a$, no bulbs will be sold to gardeners. So long as any bulbs are being planted, the price of bulbs will have to satisfy $Q_t = (1 + r)(Q_{t-1} + c) / \gamma$.
- So long as there are no sales to gardeners, the total available stock of bulbs is $Y_t = \gamma^t$.
- Assume $\gamma > (1 + r)$. Then the equation for $Q$ implies that $Q$ converges exponentially to its steady state value of $c(1 + r) / \gamma$. Once the price sinks below $a$, some of the stock is sold each period, so the exponential growth of $Y$ ceases, and $Y$ converges to its steady state value of $a - bc(1 + r) / \gamma$.

3. DETERMINING $Q_0$

- What we’ve done so far tells us how to take any initial $Q_0$ and compute a time path for $Q$ and $Y$. The $Q$ path will always converge to the steady state value. The higher the initial $Q$, the longer $Y$ grows exponentially.
- If $Q_0$ is too high, there is too large a stock of bulbs available when the price finally gets below $a$, and not enough are sold to prevent $Y$ from continuing to grow indefinitely.
- If $Q_0$ is too low, $Y$ is too small when $Q$ falls below $a$, and there is not enough stock to satisfy demand.
There is only one initial value for $Q$ that makes $Y$ converge to a finite value.

4. Examples

```r
tulips <- function(Q,nt) {
  ## demand: \( P = a - b \times Y \)
  a <- 10
  b <- .01
  c <- .5
  gam <- 3
  r <- .05
  q <- rep(0,nt)
y <- rep(0,nt)
y[1] <- 1
q[1] <- Q
for (it in 2:nt) {
  q[it] <- (q[it-1]+c)*(1+r)/gam
  if (q[it] > a) {
    y[it] <- gam * y[it-1]
  } else {
    y[it] <- gam * (y[it-1] - (a - q[it]) / b)
  }
}
return(ts(cbind(y,q)))
}
```

5. Numbers

- steady state $Q$ is 2.1
- steady state $y$ is 3423
- steady state sales to gardeners is 790
Initial Q 3475
Initial Q 3400

Time
10. How market participants must reason

- They have to guess conditions many seasons ahead, when there are finally enough bulbs to bring to market.
- Small changes in the demand curve, the interest rate $r$, or the reproduction rate $\gamma$ have big effects on the current price.
- So it is easy to see why seemingly minor bits of information can have big effects on current prices.
- Market participants with modestly different views about demand or $\gamma$ may have sharply different views about the appropriate current price, and thus be ready to enter deals that amount to “bets”.
- Bets via lending, futures contracts.
11. Variants to think about

- How much difference would it make if the owner of the first bulb of this variety could patent it, thereby obtaining a monopoly on it?
- What happens if a bulb that was previously in steady state, with constant price and sales, but the interest rate increases?
- What if the interest rate increase is so big that now \((1 + r)/\gamma > 1\)?