Econ. 487a Fall 1998 C.Sims

Discussion of FTPL Exercise and New Assignment

The FTPL exercise results were disappointing. Many people had the model's price-determination mechanism backwards—the initial price level is determinate when the model reduces to an *unstable* equation in b, not a stable one. Even those who had this straight for the most part did not go on to discuss what happens to prices after the initial period.

1. New Assignment

I said in class that we would read about and discuss banking in the US in the 19th century at our next meeting. However, the thin discussion at our last meeting and the confusion on the exercise has changed my mind. We need further discussion of FTPL, and you should reread the article on FTPL and EMU and the sections of the "Fiscal Foundations of Price Stability in..." that were assigned last time. The articles are at

`http://www.econ.yale.edu/ sims/pub/Amsterdam/EMU.pdf`

and

`http://www.econ.yale.edu/ sims/pub/hongkong/hkbrown.pdf`

I am hoping that the further discussion of FTPL will not take up the whole session next time. A good subject for more down to earth discussion is the World Bank's critique of the IMF, which is briefly described in the following New York Times article, which I think you can access without charge.

`http://www.nytimes.com/library/world/global/120398world-bank-review.html`

Try to be prepared to present an argument *against* the World Bank critique, i.e. an argument defending the IMF.

2. Answer to the FTPL exercise

The budget constraint written in terms of $b = B/P$ and $\rho = r - \hat{\dot{P}}/P$ is

$$\dot{b} = \left(\rho + \frac{\hat{\dot{P}}}{P} - \frac{\dot{P}}{P}\right) b - \tau \,. \tag{1}$$

Because in the models we looked at in class and in the assigned papers we assume C constant, the first order conditions for an optimum include a requirement that $\beta = \rho$ always. Along a perfect foresight path, $\hat{\dot{P}} = \dot{P}$. This lets us reduce (1) to

$$\dot{b} = \beta b - \tau \,. \tag{2}$$

Using the two policy equations you were given to eliminate r and τ from (2) produces

$$\dot{b} = (\beta - \phi_1 - \phi_2\theta_1)b + \phi_0 - \phi_2\theta_0 \,. \tag{3}$$

This equation is stable if $\phi_1 + \phi_2\theta_1 > \beta$, and explosively unstable if $\phi_1 + \phi_2\theta_1 < \beta$. It has a steady state, in which b remains constant, if $b = \bar{b}$, where

$$\bar{b} = \frac{\phi_0 - \phi_2\theta_0}{\phi_1 + \phi_2\theta_1 - \beta} \,. \tag{4}$$

At any initial date t_0, the amount of nominal debt $B(t_0)$ is given by history, because B can only change continuously as the government runs deficits or surpluses. If (3) is unstable, the only time path of b that is possible is the one on which b is constant at the steady-state value from (4). Any other value of b would imply explosive upward or downward movement in wealth. Upward explosion is inconsistent with optimization, since at the same time C is constant. A rational agent would eventually use some of her growing wealth to raise the level of C. Downward explosion leads to negative B in finite time, which has been ruled out as not feasible.

Now since $b(t_0) = \bar{b}$ and $B(t_0)$ is given by history, we can determine the unique $P(t_0)$ consistent with equilibrium as

$$P(t_0) = \frac{B(t_0)}{\bar{b}} \,. \tag{5}$$

In principle it might still be possible that P's after the initial date are indeterminate. However, we can determine r from the interest rate policy equation as $r = \theta_0 + \theta_1\bar{b}$, while we have from the definition of ρ and the fact that $\rho = \beta$,

$$r = \beta + \frac{\dot{\hat{P}}}{P} \,. \tag{6}$$

Therefore we can solve to find

$$\frac{\dot{\hat{P}}}{P} = \theta_0 + \theta_1\bar{b} - \beta \,. \tag{7}$$

Thus both initial P and P's rate of change are pinned down by the model, making the whole time path unique.

If instead $\phi_1 + \phi_2\theta_1 > \beta$, the b equation is stable, and every initial b implies a smooth return of b to $\bar{b}$. Thus b is not uniquely determined, and therefore P is not either.