Lending to Make Bets
with answer to exercise due 10/12

1. The assignment

Expected utility for agent i in the problem is

$$E\left[C_i - \frac{\gamma_i}{2}C_i^2\right] = (1-x_i)\mu_i + x_iQ - \frac{\gamma_i}{2}\left(((1-x_i)\mu_i + x_iQ)^2 + (1-x_i)^2\sigma^2\right) . \tag{1}$$

Note that this expression allows μ to vary with i, which was not a part of the problem you solved. After presenting the answer to the problem, in which γ (risk aversion) varies between the two types of agents but μ does not, we will look at the reverse situation. Asset trading and pricing look rather similar in the two cases, reflecting the difficulty of telling the difference between risk-trading and trading that reflects different beliefs about returns—in effect, betting.

The expression above in (1) is the answer to part (a). To answer (b), we just set $x_i = 0$ and evaluate (1) for $\gamma_i = 1$ and $\gamma_i = 2$, with $\mu = .1$ and $\sigma^2 = .01$ as specified in the problem. The result is utility of .09 for the less risk averse agent 1 and .08 for the more risk averse agent 2.

Differentiating (1) with respect to x_i and setting the result to 0 produces

$$-\mu_i + Q - \gamma_i\big((1-x_i)\mu_i + Qx_i\big)(-\mu_i + Q) + \gamma_i\sigma^2 \cdot (1-x_i) = 0 . \tag{2}$$

Solving this for x_i produces

$$x_i = \frac{(1-\mu_i\gamma_i)(-\mu_i + Q) + \sigma^2\gamma_i}{((Q-\mu_i)^2 + \sigma^2)\,\gamma_i} . \tag{3}$$

Figure 1 shows x_1 and x_2 as functions of Q, and Figure 2 shows their sum. Market equilibrium occurs where $x_1 = -x_2$, which we can see from the graph (the arrow points to it) occurs at approximately $Q = .08$. The equilibrium condition implies

$$\begin{aligned}\gamma_1 \cdot \left((Q-\mu_1)^2 + \sigma^2\right)&\left((1-\mu_2\gamma_2)(Q-\mu_2) + \gamma_2\sigma^2\right) \\ &= -\gamma_2 \cdot \left((Q-\mu_2)^2 + \sigma^2\right)\left((1-\mu_1\gamma_1)(Q-\mu_1) + \gamma_1\sigma^2\right) .\end{aligned} \tag{4}$$

For the assigned problem, the fact that μ_i does not change with i makes the quadratic factors on the left and right of (4) cancel, so that the equation becomes linear and is easily solved to yield $Q = .084615$. This seems reasonable. The asset delivers an expected total return of .1, so its value is close to .1, but it is risky, so it is worth a bit less than an asset that pays .1 with certainty. Substituting this value of Q into (3), we get $x_1 = -.376$, and it can be checked that this is just minus what we get by calculating x_2 from the same formula.

This value of x_1 represents a lot of trading. The agents begin with one unit of the asset apiece, and they trade nearly 40% of their initial holdings. To check utility levels,

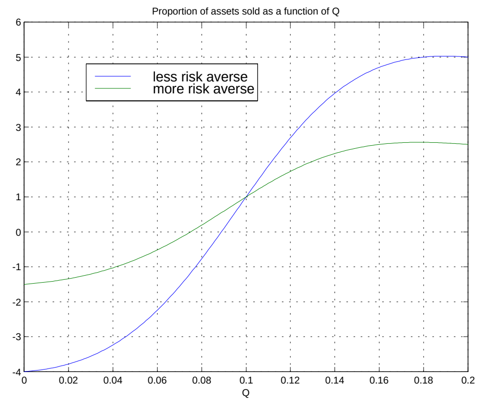

FIGURE 1.

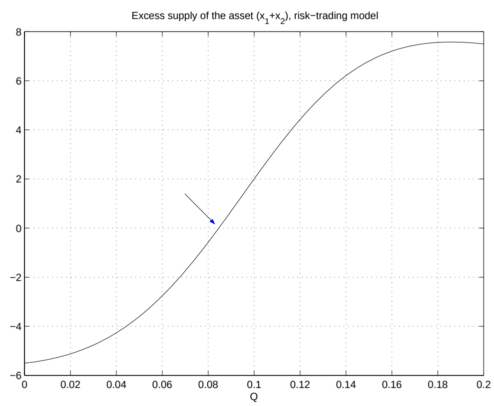

FIGURE 2

we substitute our calculated values of Q and x_i into (1), obtaining 0.090723 for agent 1 and 0.081445 for agent 2. These gains are small in comparison to the initial utility levels of .09 and .08. This completes the answer to the assigned problem.

2. Betting

If we consider instead the case where $\gamma_1 = \gamma_2 = 1$, so there is no difference in risk aversion, but $\mu_1 = .11$, $\mu_2 = .10$, we will have a case where trade is based on differences in beliefs—differences between the two types of agent in the probability distributions they assign to z, while risk aversion is the same across agents. We can still use our expressions above for x_i (3) and for market equilibrium (4), to obtain Figures 3 and 4. Analytically solving for Q and x_i here is harder, because (4) now has cubic terms that do not cancel. But it is not hard to see from the graph that Q must come out to be around .09, and closer trial and error calculation shows it to be .0937. The corresponding x's are $\pm .43$. So the amount of trading that occurs with this degree of

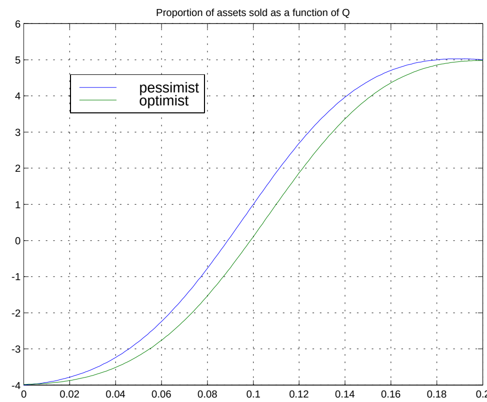

FIGURE 3

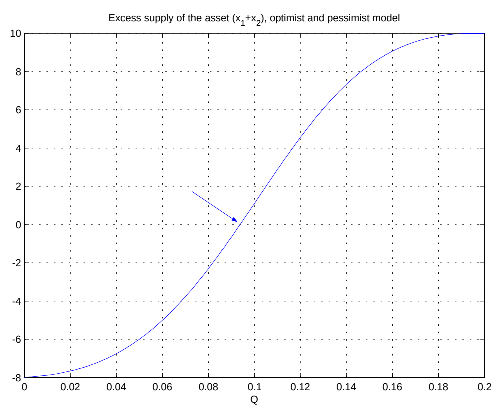

FIGURE 4

difference in beliefs is similar to what goes on with the difference in risk aversion we observed in the exercise.

If we could observe the behavior of this market under varying circumstances, we might be able to tell whether people were trading risks or betting. For example, in the risk-trading case both agents would like simply to trade in their entire endowment of risky asset for the risk free asset when Q=.1. (The two assets have the same expected return in that case, so the risk-free asset is strictly preferable.) As Q rises above that, both agents would like to short the risky asset to buy more of the higher-yielding risk-free asset. Shorting the risky asset carries as much risk as going long in it, though, so the less risk averse agent shorts the asset more aggressively.

In the optimist/pessimist model, on the other hand, the value of Q at which each agent would sell off all of his endowment of risky asset is different for each agent. The more optimistic would short the risky asset *less* aggressively than the more pessimistic agent as Q becomes high, while going long in it more aggressively when Q is low.

But since in reality we see only trades in a particular set of market circumstances, not asset demand curves as functions of Q, we can't tell from observed borrowing and lending patterns whether the motive is risk-trading or betting.

3. Work Over Next Two Weeks

As we discussed in class, we should try to stay on track with getting reading done and making progress on papers despite missing a class meeting next Monday. Each of you should make an appointment with me to discuss your paper prospectus during the period 10/20-24. There are office hours posted now for Tuesday, 10/20, (3 half-hour slots starting 10:30-11:30) and for Thursday, 10/22, (8 half-hour slots starting 2-5:30). You can write your name in a slot on the list posted on my door, or else call Diane Bowman at 432-3576 and ask her to find a time for you. If the scheduled slots fill up, I will arrange other times to see students in this class.

You are also assigned to read the section of Friedman and Schwartz's *Monetary History of the United States* on the great depression (page references on the reading list), and to read a few current articles in the financial press or on the web describing the banking situation in one of the problem Asian countries: Thailand, Indonesia, Korea, and Japan being the most prominent. Read Friedman and Schwartz with a view to understanding the differences and similarities in bank behavior between the Asian problem countries and the US in the great depression.