Answers for the "Lending to Finance Risky Investment" Exercise

In what follows numbers in parenthesis refer to the equations in the original exercise, while small roman numerals in parenthesis refer to equations on this answer sheet.

a. It is easy to see from (4)that $E[C_{I2}] = S \cdot 1.5 - RB$. Because the only random component of C_{I2} is Sz, its variance is just $S^2 \cdot 0.09$, making

$$E[C_{I2}^2] = 2.25S^2 - 3RBS + R^2B^2 + .09S^2 = 2.34S^2 - 3RBS + R^2B^2\,. \tag{i}$$

For the $j = S$ agent this is simpler, since this agent has no randomness in consumption in either period of life and the expectations therefore don't have any effect. The expected utilities are therefore

$$\text{agent I:}\quad \begin{array}{l} 1 - S + B - \frac{1}{2}(1 - S + B)^2 + \\ .9\left(1.5S - RB - \frac{1}{2}(2.34S^2 - 3RBS + R^2B^2)\right) \end{array} \tag{ii}$$

$$\text{agent S:}\quad 1 - B - \tfrac{1}{2}(1 - B)^2 + .9\left(RB - \tfrac{1}{2}R^2B^2\right) \tag{iii}$$

b. The FOC's for agent I are

$$\partial B:\quad +S - B - .9R + .9 \cdot \tfrac{3}{2}RS - .9R^2B = 0 \tag{iv}$$

$$\partial S:\quad -S + B + .9 \cdot 1.5 - .9 \cdot 2.34S + .9\tfrac{3}{2}RB = 0 \tag{v}$$

while that for agent S is

$$\partial B:\quad -B + .9R - .9R^2B = 0\,. \tag{vi}$$

c. With R given, we can solve (vi) for B, as

$$B = \frac{.9R}{1 + .9R^2} = .4662 \tag{vii}$$

Using this result in (v) gives us

$$S = \frac{.4662 + 1.35 + 1.35 \cdot 1.27 \cdot .4662}{1 + 2.106} = .8421\,. \tag{viii}$$

Then to verify that this is indeed a solution, we substitute the values of S, B and R that we have found into (iv) and check whether the equation is satisfied, which it is to within the 4-decimal-place accuracy we have been using.

d. Expected utility for agent I turns out to be .8019, and for agent S it turns out to be .7664. In autarky, agent S consumes only in period 1, achieving utility of .5, while agent 2 invests $S = .4346$, achieving expected utility of .7928. So both agents are better off than under autarky.

e. This is not a complete markets equilibrium, because agent S has nonrandom consumption in period 2, while agent I's consumption is random. Thus the ratio of marginal utilities of consumption of the two agents is not the same in all dates and states.

f. Let X be the amount of stock purchased at time 1 by agent S. Then the budget constraints for agent I are

$$C_{I1} + S - QX = W_I \tag{ix}$$

$$C_{I2} = Sz - Xz \tag{x}$$

and for agent S are

$$C_{S1} + QX = W_S \tag{xi}$$

$$C_{S2} = Xz\,. \tag{xii}$$

g. Since S and X have the same payoff to agent I in period 2, except for the change of sign, they must appear equivalent to that agent, so they must have the same coefficient in the first period budget constraint, i.e. Q=1.

h. Then agent I can be thought of as simply choosing $S - X$ just as he chose S in autarky. And with $Q = 1$ the problem for agent S is exactly the same as the autarky problem also, with X playing the role of S in the original autarky problem. So we will have $S - X = .4346 = X$ in equilibrium, and both agents will achieve exactly the autarky utility. This is an improvement over the loans-only equilibrium for S, but not for I. Here both agents have identical C's in all dates and states, so the complete-markets conditions on ratios of marginal utilities are obviously satisfied.