Econ. 487a Fall 1998 C.Sims

Lending to Finance Risky Investment

In our previous exercise, agents traded assets only to make bets or share risk. The model really had consumption only in one period, so no savings decision, in which the benefits of current versus future consumption are weighed, existed. The result was, in the "lending to share risk" setup, that the loan market was all that was needed to make markets complete. (We didn't check market completeness in the exercise. In one-good models like these, a complete markets solution is characterized by

$$\frac{U_j'\left(C_j(t)\right)}{U_i'\left(C_i(t)\right)} \tag{1}$$

being the same across all dates t, all pairs of agents i and j, and all possible random outcomes.)

Here we introduce a savings decision, so that bonds can be traded for current consumption, while giving one of two types of agents access to a risky investment technology. The result is a model with incomplete markets. Borrowing takes place, but not enough of it to finance the socially optimal amount of investment. The reason is that the investors have no market in which to shed some of the risk of large investments.

There are equal numbers of two types of agents, types $j = I, S$, where I stands for "investor" and S for "saver". Both have the same utility function over consumption in the two time periods $t = 1, 2$:

$$C_{j1} - \tfrac{1}{2}C_{j1}^2 + .9E[(C_{j2} - \tfrac{1}{2}C_{j2}^2)] \,. \tag{2}$$

For agent $j = I$, the two budget constraints (for $t = 1, 2$) are

$$C_{I1} + S = W_I + B \tag{3}$$

$$C_{I2} = Sz - RB \,, \tag{4}$$

where S is the amount invested in the risky technology at time 1, W_I is the endowment of wealth of type I agents at time 1, B is the amount borrowed by these agents at time 1, R is the (non-random) gross rate of return on loans (which we expect to emerge as close to, and probably larger than, one), and z is the random rate of return on investment. For agent $j = S$, the corresponding constraints are instead

$$C_{S1} + B = W_S \tag{5}$$

$$C_{S2} = RB \,, \tag{6}$$

where the notation should be self-explanatory. Note that by using the same notation B for both type I and type S, we are implicitly imposing market clearing.

Assume $W_j = 1$ for both types, and assume $E[z] = 1.5$, $\mathrm{Var}[z] = .09$.

a. Find $E[C_{j2}]$ and $E[C_{j2}^2]$ for $j = I, S$ as functions of S, B, and R. Use these results to find expected utility for each agent type as functions of these same three variables.

b. Find the first-order conditions for a maximum in each of the two agents' optimization problems. This means differentiating the two expected utility functions with respect to the variables the agents are choosing. Since we are looking for a *competitive equilibrium*, each agent type treats market prices—here just R—as given, and maximizes with respect to the choice variables B and (in the case of type I) S.

c. You should now have three equations in the three unknowns B, R, and S. Show that they have a solution in which $R = 1.27$, and find the corresponding levels of B, S, and $\{C_{jt}|j = I, S;\ t = 1, 2\}$. [Because these equations are nonlinear, directly solving them is a mess. Instead plug in the value of R you are given, and verify that there are S and B values that allow the equations to be satisfied.]

d. Find the expected utilities associated with this solution, and compare them to the expected utilities under autarchy ($B = 0$).

e. Show that this is not a complete-markets equilibrium.

f. Now consider the case where instead of trading bonds, the agents trade a stock with a price Q per share (in consumption good units) in the first period and with a payout of z per share in the second period. Show the budget constraints corresponding to (3) through (6) for this version of the model.

g. This stock-trading version of the model results in a competitive equilibrium in which $Q = 1$ and both agents have the same consumption in each period. Explain why. (You should be able to do this without trudging through all the calculus and algebra.)

h. Show that this stock-trading equilibrium is a complete markets equilibrium, and that in it the type I agents are worse off than in the bond-trading equilibrium.