Solutions for 9/28 Problems

1. The VAR Problem

I tried to make this a little different from the problem that was worked out in the notes, and I think ended up making it too different, so that a lot of people had trouble just getting the problem set up.

Here the yield y was not normal. The yield is the left-hand side of (2) in the notes (which gives total gross value of the investment at $t = 1$) divided by X (the amount invested at $t = 0$) minus 1. So in terms of P, B, r and X it is

$$y = \frac{P_1}{P_0}\left(1 + \frac{B}{X}\right) - e^r \frac{B}{X} - 1 \,. \tag{1}$$

You were given (in (7) in the exercise) that

$$\log\left(\frac{P_t}{P_0}\right) = W_t - W_0 + gt + \theta J_t \,. \tag{2}$$

Therefore

$$W_1 - W_0 = \log\left(\frac{P_1}{P_0}\right) - g - \theta J_1 = \log\left(\frac{y + 1 + e^r \cdot (B/X)}{1 + B/X}\right) - g - \theta J_1 \,. \tag{3}$$

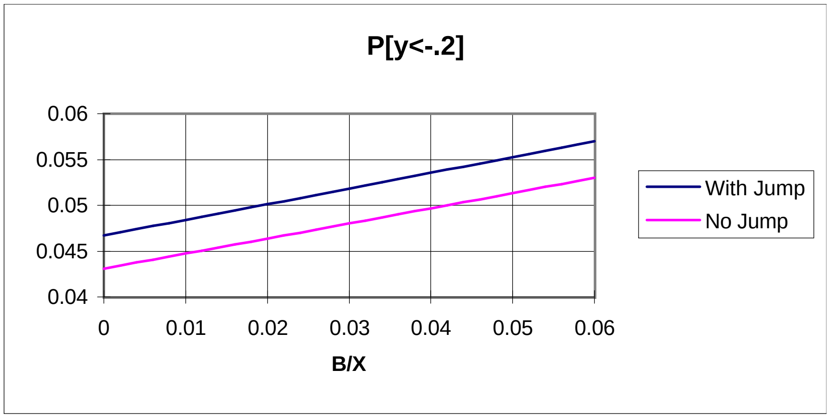
We know that $W_1 - W_0$ is distributed as $N(0, .2^2)$, $g = .12$, $r = .08$, and $\theta = .1$. The critical loss whose probabilities we are checking is $y = -.2$. We will check it for a number of different values of B/X. With all the parameters and y and J replaced by numbers, each value of B/X gives us gives us, via (3), a different value of $W_1 - W_0$ to look up in a $N(0, .04)$ table. We average these up across the three possible values of J, weighting by the probabilities of different J's, and we have our plot.

There was a typo on the formula for the cdf of the time until jump: it should have read $1 - e^{-\lambda t}$. The exercise left off the minus sign in the exponent. However, this typo should have been obvious, since it implied probabilities less than zero or greater than one. With the correction, the probability of no jump in the $(0, 1)$ interval is $e^{-.2} = .819$, and the probabilities of upward and downward jumps are both .091.

So for $B/X = 2$ and $J = -1$, for example, we get the right-hand-side of (3) as

$$\log\left(\frac{-.2 + 1 + e^{.08} \cdot 2}{1 + 2}\right) - .12 + .1 = -.0312 \,. \tag{4}$$

If we're using a $N(0, 1)$ table, we have to convert this from $N(0, .04)$ to $N(0, 1)$ by dividing by the standard error, .2, to make it -.156. The probability of a $N(0, 1)$ variable being less than -.156 is .438. For the same value of B/X, with $J = 0$, we get instead a probability of .256. And for $J = 1$ we get .124. Weighting by the three probabilities of jump, we get an overall probability of .261. This is not too different



from the probability without the jump, which is just the .256 we've already calculated. It is far above our .05 target probability, so we know we have to go to a smaller B/X to plot the region we are interested in.

The probability of a loss of 20% or more in the case of no borrowing at all ($B/X = 0$) turns out, with no jump, to be .043. With the jump, the probability of a 20% loss with $B/X = 0$ is .047, even closer to the .05 cutoff. At $B/X = .1$, the probability of 20% loss is up to .059 without jump, .065 with jump. So the overall conclusion is that for this set of parameter values, borrowing must be kept to very low levels to avoid a 5% probability of 20% loss.

A plot, made with Excel, of $P[y < -.2]$ for a range of relevant B/X values appears in the figure. In a sense here it does not matter a great deal whether the jump is taken into account or not, since the overall conclusion is that B/X must be kept very low in either case. On the other hand, the allowable amount of leverage to maintain the 5% risk is twice as great if the jump is not accounted for.

2. The Tulip Problem

Everyone seemed to get the mechanics of this right, except that several used the formula (5) in the problem handout without recalling that this formula only applies for those periods in which $Q < a/b$ (so the implied Y is positive). For part b, but not part c, of the problem, ignoring this restriction resulted in the wrong conclusion. One person calculated correctly that the fundamental value in this problem, for the part b, $Z_0 = 2$, case, is about 534, way above the initial value of $10\bar{Q} = 7.14$ that you were given in part b.

There also was some confusion about how the calculations in the exercise related to whether Q_0 was too high or too low. The part b case, where initial Z and Q implied Z going negative, implied that Q_0 was too *low*, while the reverse was true of part c. The intuition is that when Q_0 is too low, tulip owners are underestimating future demand, selling too much now at too low a price, given the returns from growing more tulips to meet future demand. Or, mathematically, one can see that the only way to get rid of the negative values of Z is to start off with a *higher* Q_0.