Econometric Implications of the Government Budget Constraint

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Carl Christ has played a leading role in reminding the profession of the elementary, but apparently easily forgotten, fact that base money creation, debt issuance, taxation, and expenditure are linked together in a single accounting identity. When he wrote about this in 1978 and 1979 and earlier (e.g. in Christ [1979]), there were econometric models in use for policy analysis that were not careful in keeping track of the implications of the government budget constraint. It was surprising to me, therefore, that his work did not create a greater stir than it did. I believe that most modelers thought of it as mattering only to the "long run," thought of their models as only appropriate for use in the "short and medium run," and therefore regarded careful treatment of the government budget constraint (GBC) as a matter of secondary importance.

This paper points out that it is still not difficult to find examples of econometric models meant for policy analysis that fail to treat the GBC carefully. It points out further that, as modelers turn increasingly to recognizing the forward-looking nature of much of economic behavior, the potential consequences of mistaken handling of the GBC grow greater and begin to slip out of the confines of the long run to menace even modelers of the short run.

How Forward-Looking Behavior Makes the GBC Affect the Short Run

The "Ricardian Equivalence Proposition" (REP) is now part of the standard vocabulary of macroeconomic discourse. There is no clearer example of the power of the GBC to affect conclusions about the short run. The apparently obvious point that a tax increase has a negative effect on private sector budget contraints turns out to be incorrect. The REP demonstrates that because of the GBC, higher taxes now mean lower taxes in the future, so that forward-looking consumers will not revise spending plans in reaction to current tax changes. In order to continue to treat the GBC as of second-order importance, macroeconomists must rely on the idea that the REP applies only in an unrealistic extreme polar case, irrelevant to the real world they are trying to model. Now there is no doubt that the REP applies only in an unrealistic extreme polar case. We do not have lump-sum taxes available. Some prices are not perfectly flexible. People do not live forever. Many people have no children or parents willing or able to provide them with financial support. These facts imply the REP is not just somewhat inexact, but probably false to a degree that is important in actual policy modeling. But this should be small comfort to modelers who ignore the GBC. There are certainly some private agents, including most holders of government bonds, who are well aware of their own intertemporal budget constraints and how they interact with the government's. Even people who don't understand or usually pay attention to these matters will eventually come to recognize the implications of an unsustainable fiscal policy. That the REP is not correct only implies that the effects on our conclusions of careful handling of the GBC may be more complicated and stronger (especially on real variables) than the simple conclusions of the REP suggest.

To illustrate this point we will consider two simple models: one of the effects on growth of income taxation and the other of the effects on the price level of fiscal policy. In the first model there are no imperfections or externalities except the fact that the only available tax is a proportional tax on value added. The model has inelastically supplied labor, so the part of this tax that is borne by labor income is non-distorting. The part borne by capital income distorts investment decisions and affects the steady-state capital stock. This is the sort of model in which the by now familiar conclusion displayed by Chamley [1986] applies: the optimal rate of taxation on capital in the long run is zero. Here the optimal rate of income taxation is zero in the long run, despite the existence of an exogenously given level of real government spending.

But the implications of Chamley-style analysis of capital taxation are often misunderstood. A more accurate characterization of the optimality result, when the GBC is taken into account, is that in the short run the tax rate should be so high that it becomes possible to make the tax rate zero in the long run. We will not analyze optimal policy directly, but instead consider the effects of unanticipated shifts in the tax rate, obtaining some perhaps unexpected conclusions.

We consider an economy in which there is no money and the single commodity is the numeraire. Interest rates on government bonds are quoted as real interest rates, therefore. There is a single representative private agent, infinitely lived, who maximizes

$$E\int_0^\infty e^{-bt}\log C_t\,dt \tag{1}$$

subject to

$$\left(qC^2+(1-q)I^2\right)^{\frac{1}{2}} = AK^a L^{1-a} \cdot (1-t) + rB - \dot{B}$$
$$\dot{K} = I - dK \tag{2}$$
$$L \equiv 1$$

C is consumption, I investment, K capital, t the tax rate, B the stock of outstanding government bonds, r the interest rate, and A an exogenously evolving level of technology.

The government chooses t and B subject to the constraint

$$\dot{B} + tY = rB + GY\,, \tag{3}$$

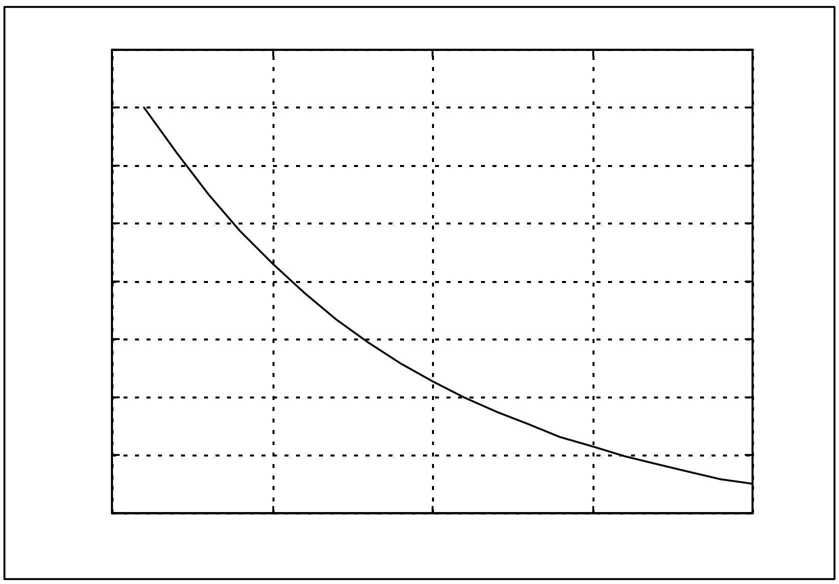
where we have defined $Y = AK^a$. The government cannot be modeled as setting t arbitrarily in this model, as if it leaves the tax rate unresponsive to the level of debt, debt will explode exponentially, violating the transversality conditions of private agents. We therefore include in the model a systematic reaction of taxes to debt, with fiscal policy actions disturbing this reaction function rather than setting paths for t directly:

$$t = -f_0 + f_1 \frac{B}{Y} + F\;. \tag{4}$$

The variables F, A, and G are all thought of as evolving exogenously, capable of jumping discontinuously or having white noise components in their derivatives, but with well-defined sample paths.

Because labor is supplied inelastically here, the component of the general income tax that falls on labor is non-distorting. An optimizing government that could tax capital and labor differently would tax only labor. But with only a proportional tax on Y available, taxation distorts investment decisions. High taxes on returns to capital discourage investment and thereby lower income. One might think, then, that a surprise increase in F, sustained forever, would lower capital stock and output. Figure 1 below shows that the sustained increase in F does not produce a sustained increase in t. Indeed after 10 years the tax rate has fallen below its original level.

Figure 1



The parameter values that produce Figure 1 are a=.4, b=.05, d=.07, f_0=.02, f_1=.15, q=.85. The dynamics are calculated from a linearization of the model about its steady state.[1]

The long run decline in the tax rate occurs because the initial increase reduces debt. As debt declines, it lowers the endogenous component of the tax rate. There is an initial negative effect of

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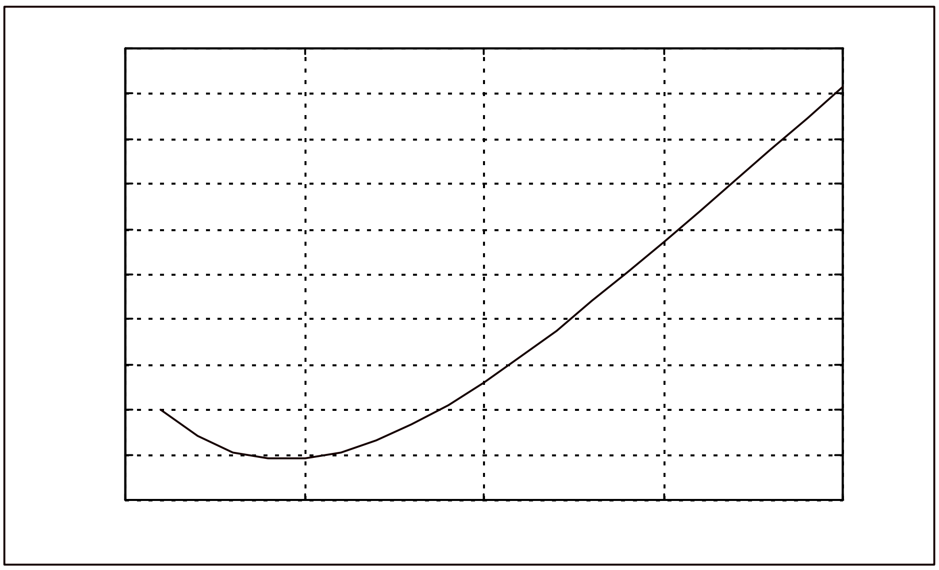
[1] The software used for solving the linearized systems for this and the next example is available as a set of Matlab m-files in ftp/pub/sims/gensys at the econ.yale.edu ftp-anonymous site.

the rise in F on investment, but because anticipated future taxes are more important than current taxes for investment decisions, the negative effect wears off long before the tax rate has returned to its original level. The dynamic path of the capital stock's response is shown in Figure 2.

Figure 2



Note that investment has risen above its initial level 5 years after the initial disturbance, and the stock of capital itself is above its initial level two years before the tax rate has come back down to its initial level. This result is fairly robust to the f coefficients in the fiscal rule. It is true that a high f_1 favors a rapid recovery of investment after an F increase by making taxes drop quickly as debt drops, but the debt itself drops more quickly when f_1 is small.

In the next example model, we see that tax policy can have direct impacts on prices. We postulate a simple economy in which the real interest rate is constant, i.e.

$$dp = (r - \bar{r}) \cdot dt + dh \ , \tag{5}$$

where r is the nominal rate, $\bar{r}$ is the real rate, p is the log of the price level, and h is an endogenously determined martingale process. We postulate also a demand for money equation

$$m - p = -\tfrac{1}{2}\log(r) + e_d \ , \tag{6}$$

where m is the log of the money stock and e_d is an exogenous martingale process. Monetary policy is taken to follow the reaction function

$$dm = a \cdot r \cdot dt + de_m \ , \tag{7}$$

where e_m is an exogenous martingale process.

Macroeconomists have been used to analyzing systems of equations in *r*, *m*, and *p* like (5)-(7) as if they were capable by themselves of determining the price level. It is not hard to show that they do determine a unique price level if explosive time paths for the variables in the system can be ruled out and $a < 1$, and macroeconomists have often proceeded as if it were generally reasonable to rule out explosive solutions to such systems. But as a previous paper of mine (Sims

[1994]) showed,[2] when equations like these are put in context of a full general equilibrium model, it may not be possible to rule out explosive paths. That paper pointed out the existence of cases, under certain assumptions about the transactions technology, in which the price level is not uniquely determined even when money stock is held fixed.

Nonetheless under a range of assumptions on the transactions technology, the unique stable solution to a system like (5)-(7) is the only solution. Of course a complete model would include the government budget constraint and a tax policy rule, for example

$$dt = f \cdot db + de_f \tag{8}$$

$$(dp + db)e^{b} + dm \cdot e^{m-p} + t = r \cdot b \tag{9}$$

where b is real government interest-bearing debt, b is the discount factor in a representative agent's utility function, and t is the real level of lump-sum taxes. Equation (8) is the tax policy rule, while (9) is the government budget constraint. If prices are to be stable in equilibrium, (8) must have $f > b$ to avoid exploding real debt, but so long as this condition is met, debt will be stable and the fiscal policy disturbance e_f will have no effect on prices at all. A stable, uniquely determined price level is also possible when $a > 1$ and $f \leq 0$, as was pointed out by Leeper (on the assumption that unstable paths could be ruled out) and my paper Sims [1994] (more

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[2] Obstfeld and Rogoff [1983] made the same point much earlier in a somewhat different type of model.

generally). In these cases fiscal policy does affect the price level, but in some sense only because monetary policy is passive -- m cannot be fixed in these equilibria (because that would violate $a > 1$). When $a < 1$ and $0 < f < b$, an interesting case arises. Stable prices then imply debt exploding more slowly than e^{bt}. Since taxes, because of (8), explode in proportion to debt, it turns out that the slow explosion of the real debt is not inconsistent with transversality conditions, even though explosion at a similar rate of real money balances would violate transversality. In such an equilibrium, the effects of monetary and fiscal policy on the price level are qualitatively similar to those in equilibria with $a < 1$ and $f > b$.[3]

To make this point concrete, we compare the responses of prices to e_f and e_m for two nearly adjacent points in the parameter space: $a = .98, f = .03$ and $a = 1.02, f = -.03$. The former of these is what Woodford calls "Ricardian", as it makes the impact of e_f on prices zero. Figure 3 shows the effects on the log of prices, the log of the money stock, and the interest rate over eight years (32 quarters) following "unit" shocks to e_m. Note that the responses are much

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[3] Recently Woodford [1995] has pointed out that besides the cases in which fiscal policy is irrelevant to the price level or in which money is passive, there are also cases in which m remains fixed, the price level is uniquely determined, and fiscal policy has large impacts on the price level. These are cases where with a simple $a < 1$, $f > b$ policy the price level is not uniquely determined because speculative hyperinflations are possible. With certain choices of fiscal policy parameters, the fiscal policy picks out a unique equilibrium path corresponding to what otherwise would be one of a continuum of speculative hyperinflationary equilibria.

stronger in the active-money or Ricardian regime. The absolute scales of the graphs are arbitrary, but the comparison across graphs is valid: they represent responses to the same size newly created wedge between $\dot{m}$ and ar. Figure 4 shows the effects of fiscal policy on the same variables in the active-fiscal regime. (For the active-money regime, all the effects are zero.) Note that the qualitative character of the responses matches those for the active-money responses to monetary policy; shocks to tax policy have effects quite similar to shocks in monetary policy under the active-money regime. Figure 5 shows the responses to money-demand shocks. The active-money regime makes these shocks purely neutral, being immediately and entirely absorbed in a one-time shift in the price level. The active-fiscal regime makes the effect on the price level somewhat smaller, but generates persistent effects on the interest rate and inflation that are not there under the active-money regime.

Clearly the behavior of the model changes discontinuously between the two nearby parameter points we have analyzed here. This kind of discontinuity will be pervasive in models that include both fiscal and monetary policy reaction functions, and it makes for difficult numerical problems in estimating them or even in simulating them. Optimization routines exploring the likelihood of dynamic models are likely to be stymied or slowed down by such discontinuities. Attempts to solve and simulate these models are likely to be frustrating because similar, apparently realistic parameter values may lead to very different model behavior, and it is difficult to determine a priori where the regions of discontinuity in the parameter space may lie.

The GBC in Some Recent Models

Two modern policy-oriented models that have been laid out and discussed at book length are John Taylor's [1993] and that of McKibbin and Sachs [1991]. Both these models represent efforts to incorporate forward-looking behavior explicitly into policy models. Both sacrifice statistical credibility in favor of hoped-for advantages in internal model coherence from the use of forward-looking behavior. Taylor uses some instrumental variables estimation, but does not use it in every instance that the model implies it is necessary. Though he compares the time series properties of a small example model in the first part of his book to those of a reduced form model, there is no comparably detailed analysis of fit for his full-scale model. McKibbin and Sachs forego explicit statistical inference entirely and present as evidence of fit only "tracking" exercises, in which certain variables taken as exogenous (for example the money supply) are adjusted informally to bring the model's simulated paths into line with observations. The money supply numbers that emerge are not shown, on the ground that they reflect velocity shocks not allowed for in the model. The exercise is not logically vacuous, because more data are being matched than exogenous factors are being backed out of the calculations. On the other hand, because of its partially ad hoc nature and the lack of a description detailed enough to make it reproducible, it is almost weightless as evidence of model fit.

But the main focus of our discussion here is on treatment of the role of the GBC in policy analysis. From the standpoint of analyzing the effects of a tax change as in the first example of the preceding section, the McKibbin-Sachs model is close to being usable. It has a tax reaction function (not estimated, but postulated) that keeps real debt stable, and it has a version of a government budget constraint. The tax reaction displayed in the book has only a lump-sum tax, treated as part of labor income, responding to debt, but it would not be complicated to add a tax-reaction function that acted on rates of distorting taxes as well.

From the standpoint of analyzing the interaction of monetary and fiscal policy as in the second example, however, the McKibbin-Sachs model has serious deficiencies. It includes a government budget constraint written in real terms -- exactly as in equation (3) of our example model above. But our example model ignores inflation and price level determination. If our model included a price level and money, we would have to rewrite (3) as

$$\frac{d}{dt}\frac{B}{P}+\frac{d}{dt}\frac{M}{P}+\frac{\dot{P}}{P}\cdot\left(\frac{B}{P}+\frac{M}{P}\right)+tY=\left(r+\frac{\hat{\dot{P}}}{P}\right)\frac{B}{P}+GY\ . \tag{10}$$

McKibbin and Sachs use (3) instead and thereby drop the terms involving M and omit any distinction between $\dot{P}$ and $\hat{\dot{P}}$ -- that is between the actual historical inflation rate and the rate expected to prevail from the current date onward. In an advanced economy with large debt and a well-developed tax system, the real value of money balances is a small part of real government debt and seignorage revenue is a small part of total revenue. Therefore dropping the terms in M is justifiable as an approximation. But this leaves the model's failure to make a distinction between backward-looking measures and forward-looking measures of the inflation rate. If the price level moves downward in a discontinuous, unanticipated jump, then, if real taxes and the nominal interest rate are unchanged, real debt jumps upward discontinuously. Equation (3) pretends that this is not true. The McKibbin-Sachs model therefore ignores an important source of real effects of fiscal policy -- the losses to holders of bonds from unanticipated inflation. In a model that shows Ricardian equivalence, this would not matter much. In the McKibbin-Sachs model, however, it matters a lot. Indeed even when Ricardian equivalence does hold, when there is cross-country holding of national debts, differences in unanticipated inflation across countries generate substantial international wealth transfers, a fact that the McKibbin-Sachs formulation assumes away.[4]

The model of Leeper and Sims [1994] includes an accurate government budget constraint, as well as estimated monetary and fiscal reaction functions. As is described in that paper, the discontinuities in the likelihood did cause problems for that model, and it is not as detailed as McKibben-Sachs. Unlike either McKibben-Sachs or Taylor's model the Leeper-Sims model sticks to a closed economy formulation.

Thus all three of these models fall short of the goal of simultaneously fitting the data well, being of the scale required for policy analysis in an internationally linked economy, and consistently linking the intertemporal implications of the government budget constraint to forward-looking private sector behavior. Nonetheless, taken as a group, they suggest that the goal may not be too far ahead of us.

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[4] The McKibbin-Sachs model is in discrete time. It has real debt *B* changing only when the real deficit *DEF* is non-zero, and *DEF* is defined as real government expenditures less real taxes less the real interest rate (the nominal rate less expected inflation) times the level of real debt. But in reality *DEF* can remain fixed while the price level changes. The real value of the debt would then change without non-zero *DEF*.

REFERENCES

Chamley, C. [1986]. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica* 54, 607-22.

Christ, Carl [1979]. "On Fiscal and Monetary Policies and the Government Budget Constraint," *American Economic Review* 69, 526-38.

Leeper, Eric M. (1991), "Equilibria Under `Active' and `Passive' Monetary And Fiscal Policies," *Journal of Monetary Economics* 27, February, 129-47.

Leeper, Eric M. and Christopher A. Sims [1994]. "Toward a Modern Macroeconomic Model Usable for Policy Analysis," *NBER Macroeconomics Annual*

McKibben, Warwick J. and Jeffrey D. Sachs [1991]. *Global Linkages* (Washington, DC: Brookings Institution).

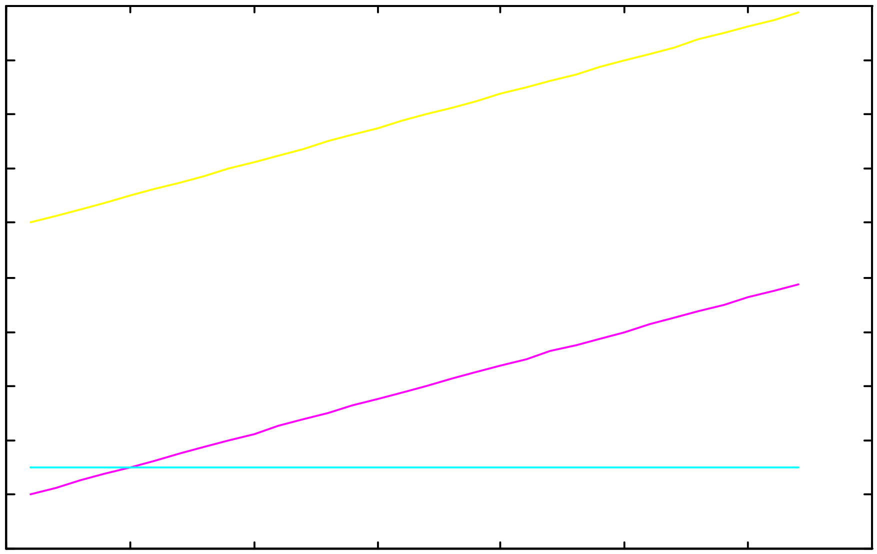
Obstfeld, M. and K. Rogoff [1983]. "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy* 91, 675-87.

Sims, Christopher [1994]. "A Simple Model for the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory* 4, 381-99.

Taylor, John [1993]. *Macroeconomic Policy in a World Economy*, (New York, London: W.W. Norton).

Woodford, Michael [1995]. "Price Level Determinacy without Control of a Monetary Aggregate," NBER Working Paper No. 5204.

Figure 3



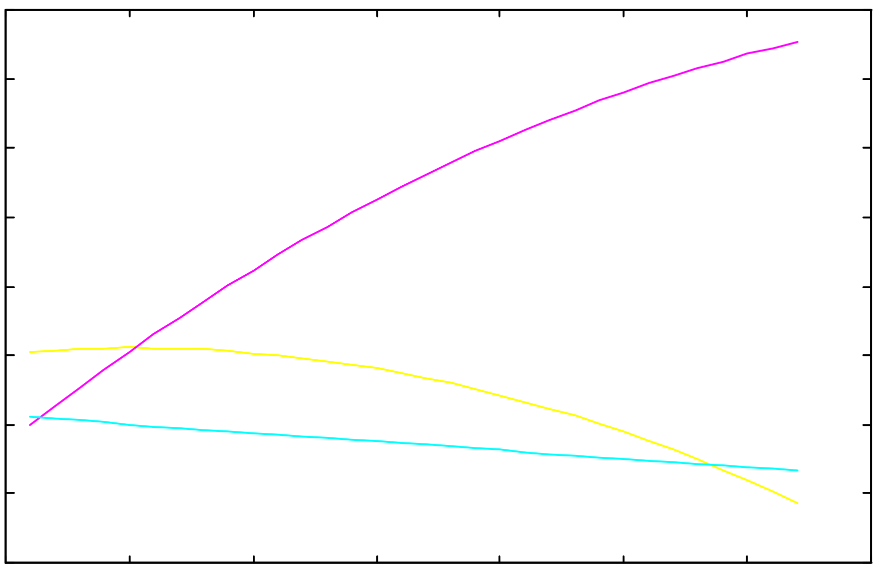


Figure 4

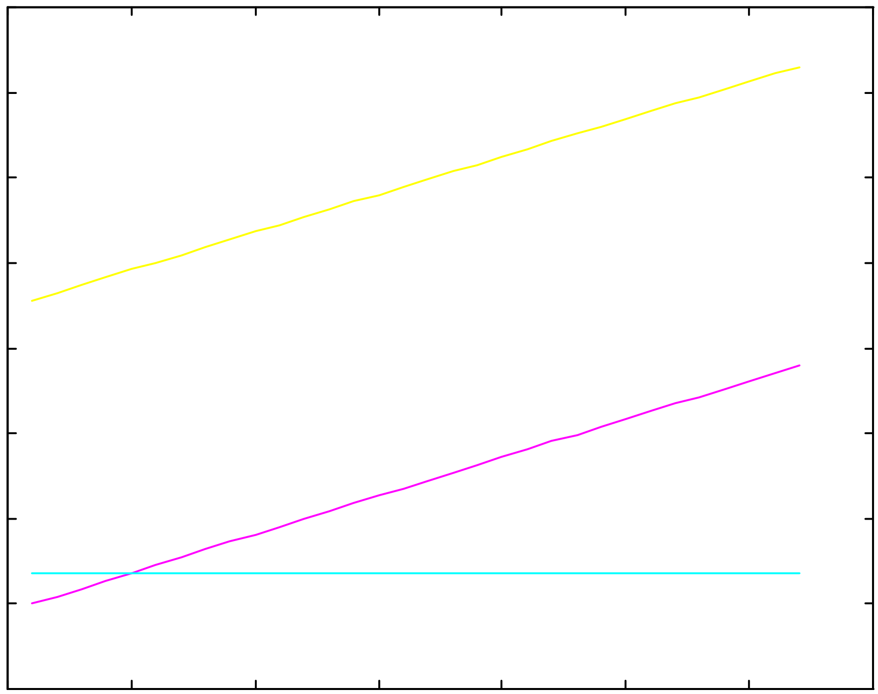


Figure 5