Econ. 526b Spring 1998 C. Sims

Extension of Model Solving Exercise

As we jointly discovered, the previous model-solving exercise was close to impossible, because the use of simulated sample paths to generate the fit criterion was a bad approach to the problem. In this highly non-stationary model, the simulated solutions quickly approached a region near $C = 1$, where the Euler equation residuals were all very small. Only a few simulated observations in each repetition were affecting the criterion function, which was therefore ill-behaved. I have since solved the model with a Galerkin method and Monte Carlo integration and will discuss this with you in class. Analytic integration, though possible in this problem, is a forbidding computational task. The analytic integration has to be done separately over as many as four subintervals, with the number and nature of the intervals varying with the state and the parameters of the approximating polynomial.

A much more straightforward task, that I completed in about one hour, starting from the code I wrote for the previous intractable problem, is to solve the standard one-state growth model of the Taylor-Uhlig symposium. The model is of a representative agent who solves

$$\max_{\{C_t, K_t\}_{t=0}^{\infty}} E\left[\sum_{t=0}^{\infty} \beta^t \frac{C^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

subject to

$$C_t + K_t = A_t K_{t-1}^{\alpha} + \delta K_{t-1} \ . \tag{2}$$

We do not need to separately assume $C, K > 0$ because the technology and utility function guarantee that the agent can and will always keep both C and K positive.

The FOC is

$$C_t^{-\gamma} = \beta E_t\left[C_{t+1}^{-\gamma} \cdot \left(\alpha A_{t+1} K_t^{\alpha-1} + \delta K_t\right)\right] \ . \tag{3}$$

Assume

$\alpha = .3$

$\beta = .95$

$\gamma = 2$

$\delta = .93$

A_t i.i.d. with $\log A_t \sim N(0, .01)$.

To make this a problem with a single state variable, we must define the state as

$$W_t = A_t K_{t-1}^{\alpha} + \delta K_{t-1} \ . \tag{4}$$

Then the technology (2) can be written as

$$W_t = A_t \cdot \left(W_{t-1} - C_{t-1}\right)^{\alpha} + \delta\left(W_{t-1} - C_{t-1}\right) \ . \tag{5}$$

Parameterize the decision rule, which gives optimal C as a function of W, as

$$C^*\left(W\right) = e^{\phi_0 + \phi_1 W + \phi_2 W^2} \ . \tag{6}$$

Start your simulations with $K_{-1} = .5$, and run them for 80 periods, with at least 100 replications. Use the cross product of your Euler equation residuals with a constant vector, lagged W, and lagged W^2 to generate your discrepancy vector. Plot the Euler equation residuals and check them for serial correlation as an accuracy check. Also repeat your estimates with a new draw of the shocks, to check for the size of Monte Carlo sampling error. To avoid the equation-solver's drifting toward a trivial non-solution in which it makes all C's extremely large, define the Euler equation residuals as

$$\eta_t = \beta\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\left(A_t K_{t-1}^{\alpha} + \delta K_{t-1}\right) - 1 \ , \tag{7}$$

i.e. as what one gets by dividing (3) through by its left-hand side.