EXERCISE ON CONTINUOUS TIME FTPL AND RATIONAL INATTENTION

(1) **Balance sheets and inflation control** Consider an economy in which the representative agent solves

$$\max_{C,M,B} E\left[\int_0^\infty e^{-\beta t}\log(C_t)\,dt\right] \tag{1}$$

subject to

$$C_t \cdot (1+\gamma v) + \frac{\dot{B}+\dot{M}}{P} = Y + \frac{rB}{P} - \tau\,. \tag{2}$$

Here $v = PC/M$. The central bank has the budget constraint

$$\frac{\dot{M}-\dot{B}_C}{P} = \frac{rB}{P} + \tau_C\,. \tag{3}$$

τ_C is the rate (in real terms) at which the bank makes transfers to the fiscal authority. It is natural to suppose that it is positive, enough to keep the bank's excess of assets over liabilities small, when there is net positive seignorage. The treasury has the budget constraint

$$\frac{\dot{B}_F}{P} + \tau + \tau_C = \frac{B_F}{P}\,. \tag{4}$$

Market clearing requires that debt issued by the treasury be the sum of debt held by the central bank and debt held by the public, i.e.

$$B_F = B_C + B\,. \tag{5}$$

(a) Suppose the monetary policy has been to make M grow at a constant rate of $-\frac{1}{2}\%$ per year, that this policy has been anticipated by the public and expected to go on forever. Show that, if we assume constant Y and accommodating fiscal policy, the economy has a steady-state equilibrium with constant r, C, and $\pi = \dot{P}/P$.

(b) Show that with fiscal policy setting $\tau + \tau_C = 2\beta B/P$, this equilibrium is not unique. There are also equilibria in which v explodes to infinity (but none in which it goes to zero).

(c) **Extra credit.** Show that the explosive equilibria make money balances go to zero in finite time.

(d) Suppose the monetary authority decides, as a surprise to the public, to change the money growth from $-\frac{1}{2}\%$ per year to 3% per year and keep the new higher rate of money growth in place forever. Assume the public immediately understands and believes in the permanence of the new policy. Assume $\gamma = .00018$, which is roughly right for the US, based on the ratio of high-powered money (unadjusted for reserve requirements) to nominal GDP and the current nominal interest rate. Also assume that the discount rate $\beta = .02$. Show that the rise in the inflation rate may require the bank to instantaneously absorb money balances by selling off some of its bond holdings, if it is to implement the policy without an infusion of capital from the treasury and without allowing a jump in the price level. What is the minimum ratio of marketable bonds to money (assuming all its liabilities are money) that it will require to implement the changed policy without a jump in the price level?

(2) **Simple RI problem** Consider the problem of choosing X_t to minimize $E_t[(Y_t - X_t)^2]$. The problem starts at $t = 0$, when the optimizer has as an initial probability distribution $Y_0 \sim N(\hat{Y}_0, \sigma_0^2)$ and must choose X_0 given that distribution. The optimizer knows that $Y_t = Y_{t-1} + \varepsilon_t$, all t, where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ for all t and ε_t is independent of all values of Y_s and ε_s for $s < t$. At $t = 1$ the optimizer can reduce his uncertainty about Y_1 before choosing X_1, subject to a constraint that the mutual information between X_1 and Y_1 conditional on information at $t = 0$ be less than or equal to κ bits. This process continues indefinitely, with the optimizer at each t being able to improve his knowledge of Y_t subject to the same form of mutual information constraint, and with the same κ.

(a) Show that the agent will choose to make the conditional distribution of Y_t given information at t normal at every date.

(b) Derive an equation that shows how σ_t^2, the conditional variance of Y_t given information at t, evolves over time, and find its steady state value as a function of κ and σ_ε^2.