(A) (60 points) This question concerns central bank balance sheets and control of inflation.

(1) (7 points) Central banks that attempt to control the exchange rate can find themselves with large amounts of domestic currency (peso) denominated liabilities as well as large amounts of foreign currency (dollar) denominated assets. Explain what kind of sequence of policy actions and/or non-policy events can lead to this situation.

Central banks that attempt to keep domestic currency from appreciating rapidly will sell domestic currency for foreign currency. This will by itself tend to increase the domestic money stock. To prevent this, the bank may "sterilize", meaning that it sells interest-bearing domestic currency bonds for domestic currency. The net outcome is an increase in foreign-currency assets and a decrease in domestic-currency assets. If the intervention is sustained and large enough, the bank may run out of domestic-currency denominated bonds to sell. At that point, to continue sterilizing it will have to borrow in domestic currency. This is often done by Central bank issue of nominal bonds.

(2) (5 points) Explain how this situation can lead to the central bank's having negative net worth, measured as market value of assets minus market value of liabilities.

Exchange rates are notoriously volatile. If dollar-denominated assets become a large part of the banks total assets, and if the domestic currency appreciates, the bank's liabilities can easily start to exceed its assets, at market value. Also, domestic currency denominated bonds often are seen as subject to exchange risk, so they pay a higher rate of interest even during times when the exchange rate is stable. This can make interest earnings minus interest payment obligations negative, if there are large amounts of central bank peso bonds outstanding. Over time, this negative seignorage can lead to negative net worth, even if net worth starts positive and the exchange rate is stable.

(3) (20 points) Here is a model of a simple flex-price economy with a central bank that holds dollar bonds and issues peso bonds as well as non-interest-bearing peso money.

Private agents

$$\max_{C,B,M} E\left[\sum_{t=0}^{\infty} \beta^t \log(C_t)\right] \quad \text{subject to} \tag{A1}$$

$$C_t(1+\gamma v_t) + \frac{B_t + M_t}{P_t} = \frac{R_{t-1}B_{t-1} + M_{t-1}}{P_t} + Y_t - \tau_t \tag{A2}$$

$$v_t = \frac{P_t C_t}{M_t} \tag{A3}$$

Central bank

$$\text{Budget constraint}: \quad F_t - \frac{B_t^C + M_t}{P_t} = \rho F_{t-1} - \frac{R_{t-1}B_{t-1}^C + M_{t-1}}{P_t} - \sigma_t \tag{A4}$$

$$\text{Monetary policy}: \quad M_t = \phi M_{t-1} \tag{A5}$$

$$\text{Portfolio policy}: \quad F_t = F_{t-1} \tag{A6}$$

Treasury

$$\text{Budget constraint}: \quad \frac{B_t^T}{P_t} = R_{t-1}\frac{B_{t-1}^T}{P_t} - \tau_t - \sigma_t \tag{A7}$$

$$\text{Fiscal policy}: \quad \tau_t = -\theta_0 + \theta_1 \frac{R_{t-1}B_{t-1}^T}{P_t} \, . \tag{A8}$$

Variable names

C: consumption
B^C: CB issued peso bonds
M: money
R: gross nominal interest rate
Y: endowment
F: CB holdings of dollar bonds
σ: seignorage payment from CB to treasury
B^T: Treasury issued peso bonds
B: Privately held bonds
v: velocity
P: price level
τ: lump-sum taxes
ρ: gross real rate on dollar bonds

Assume $\theta_0 > 0$, $\theta_1 > \beta^{-1} - 1$, $\gamma > 0$, $\phi > 0$. Also assume that, when the central bank's net worth is negative, it turns over no earnings to the Treasury ($\sigma_t = 0$), while if its net worth is positive σ_t is chosen to keep net worth of the central bank in dollars constant. Market clearing conditions are implicit in the use of the same variables in different agents' equations, except $B^C + B^T = B$, the bond market clearing condition.

Show that, if net worth of the central bank begins as positive and fluctuations in Y_t are small enough, this economy has a uniquely determined rational expectations equililbrium with constant velocity.

There was a sign error (corrected in red above) in (A4).

The private FOC's can be reduced to

$$(1 - \gamma v_t^2) = R_t^{-1}$$

$$\frac{Z_t}{M_t}(1 - \gamma v_t^2) = \beta E_t \left[\frac{Z_{t+1}}{M_{t+1}}\right] \text{ , where}$$

$$Z_t = \frac{1}{v_t(1 + 2\gamma v_t)} \, .$$

Note that Z_t is monotone decreasing in v_t and $Z_t \to \infty \Rightarrow v_t \to 0$, $Z_t \to 0 \Rightarrow v_t \to \infty$. With M growing at the fixed gross rate ϕ, the difference equation in Z becomes

$$\phi Z_t(1 - \gamma v_t^2) = \beta E_t Z_{t+1} \, .$$

This equation has a constant solution with $(1-\gamma v_t^2) \equiv \beta$, so long as $\phi > \beta$, $\beta \in (0,1)$ and $\gamma > 0$. If Z_t starts above its steady-state value, the difference equation implies that its expected value grows without bound. This implies $v_t \to 0$, which in turn is easily shown to imply $M_t/P_t \to \infty$. This can be ruled out by a transversality argument, since with large enough real balances the current-period consumption benefits of spending some of one's real balances must outweigh any negative effects on future transactions costs. (A more complete version of this transversality argument would have been welcome, but no exam answer was even as complete as this answer.) If the initial Z_t is below the steady state value, then expected Z_t must go to zero, but this would require $v_t \to \infty$. Since the right-hand side of $(\mathrm{A}3)$ cannot be negative, while the left-hand side must become negative for large enough v, such paths are not feasible.

In an equilibrium like this, if σ_t is always positive, the central bank budget constraint becomes (because policy makes F_t constant and σ_t keeps the dollar net worth of the bank constant)

$$\frac{B_t^C + M_t}{P_t} = \frac{B_{t-1}^C + M_{t-1}}{P_{t-1}}$$

This requires that

$$\sigma_t = \kappa\left(1 - \frac{P_{t-1}}{P_t}\right) + (\rho - 1)F - (R_{t-1} - 1)\frac{P_{t-1}}{P_t}\frac{B_{t-1}}{P_{t-1}} .$$

The right-hand side of this expression is inflationary capital gains or losses on total domestic liabilities (the constant κ) plus the interest earnings on foreign debt, minus the interest payments on domestic debt, If net worth is positive and inflation is not too variable, this will imply positive σ_t in all periods and is therefore certainly sustainable.

The treasury's fiscal policy implies that real treasury debt follows a stationary process. Therefore total debt does also, and consumers' TVC's will not be violated through implied debt holdings.

(4) (20 points) Show that in this model if central bank net worth begins as negative, there may be a lower bound on ϕ, the rate of growth of the money stock, below which the policy is infeasible.

Surprisingly, no one made the connection between net worth and the constraints on ϕ in this question, though most did realize that small ϕ implies low seignorage. Setting $\sigma_t = 0$ (because of negative net worth) in the CB budget constraint, dividing it through by C_t, and applying E_{t-1} to the whole equation, it becomes

$$E_{t-1}\left[\frac{B_t^C + M_t}{P_t}\right] = -(\rho - 1)F - (\beta^{-1} - \phi^{-1})\bar{v}^{-1}\beta^{-1}\frac{B_{t-1}^C + M_{t-1}}{P_{t-1}} .$$

This is an unstable difference equation with a positive steady state value. If $(B^C + M)/P$ starts out below the steady state value, it diverges downward, and this path will be arrested when the declining CB liabilities bring it into positive net worth at market value and σ_t becomes positive again. If the bank's real liabilities start out above the steady state, though, they explode upward. This violates transversality, so long as the treasury cannot create negative debt to offset the exploding central bank debt. The steady state grows lower as ϕ decreases. Therefore, at a given, sufficiently high, level of negative net worth, the CB will encounter a minimum value for ϕ below which it is cannot generate enough seignorage to pull itself out of negative net worth.

(5) (8 points) Do these results depend on the central bank's policy of keeping $F_t = F_{t-1}$? Is there a better bank portfolio policy, from the point of view of social welfare?

(No one came close to answering this correctly.) If $\beta^{-1} = \rho$, the expected return on foreign and domestic debt is the same, and modifying the bank's portfolio therefore has no effect on the expected time path of its net worth. If ρ is below β^{-1}, selling foreign-currency assets and using the proceeds to retire domestic debt improves the bank's seignorage prospects. Furthermore, in that case it improves social welfare: the bank's holding of foreign assets are an investment for the country and earnings on them are part of the resources available for consumption. With the return on this investment below β^{-1}, consumers' welfare is increased by selling the foreign debt and correspondingly increasing current consumption. All these remarks of course hold in reverse in the case of $\rho > \beta^{-1}$. These remarks assume, in accordance with the model assumptions, that there are no exchange rate effects from the portfolio adjustment. Indeed in this model, there is no point in sterilized foreign exchange interventions.