ATTACKS EXERCISE

Consider an economy in which the government issues nominal government debt and policy is to maintain a fixed real primary surplus τ and a fixed nominal interest rate r. We assume r is chosen so that it delivers a constant exchange rate S, which is the same as the price level. The economy contains an institution (e.g. a large bank) that has borrowed in foreign currency and invested in domestic-currency bonds or loans. At the current exchange rate the institution is solvent, but if the exchange rate jumps from S to S' with $S'/S > F$ the resulting loss in dollar value of the institution's assets makes it insolvent. For simplicity, we assume that in this event there is a fixed dollar value L that the government has to pay to "bail out" the institution from its insolvency.

If an attack on the exchange rate never occurs, and is understood by everyone to have zero probability of occurrence, there is a standard FTPL constant-price equilibrium with $r = \rho$.

Now we introduce the possibility that market participants believe that there can be an "attack" on the country's exchange rate. They believe, and we assume this belief is correct, that the probability of an attack over a small time unit δ is $\lambda\delta$, with λ constant. This implies that the probability that the attack does not occur between times t and $t + T$ is $e^{-\lambda T}$. For an agent holding a domestic currency bond, the expected rate of return over an interval of length δ is approximately for small δ

$$\frac{1}{\delta}\left(r\delta - \lambda\delta\frac{S}{S'}\right) , \tag{1}$$

where S' is the post-attack exchange rate and S is the pre-attack rate. the exact rate, as $\delta \to 0$, is

$$r - \lambda\frac{S}{S'} . \tag{2}$$

We assume investors are risk neutral, so that in the pre-attack period, S is constant, $r - \lambda\frac{S}{S'} = \rho$. If there is an attack, and if after the attack τ remains constant at its previous level and $r = \rho$, total government liabilities must satisfy

$$\frac{B}{S'} + L = \frac{\tau}{\rho} \tag{3}$$

Since B can't jump, S jumps at the time of the attack, and S' is determined by this equation. If the S' determined by this equation is below FS, The attack fails, and since we have rational expectationa and perfect foresight here, there is no equilibrium in which the attack occurs.

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Before the attack, the government must in order to keep inflation at zero pay an interest rate $r > \rho$ to compensate for the attack risk.

(1) Before the attack we have assumed S is constant, and τ is constant throughout. Given the parameters ρ, λ, τ and L, B/S is constant and can be calculated. What is it? [Hint: It's not τ/ρ.]

(2) What is the pre-attack value of r, again as a function of the parameters.

(3) With the other parameters held constant, are there values of λ that make $S' < FS$ (and hence rule out an equilibrium with an attack)?

(4) Again with other parameters held constant, are there levels of τ that make $S' < FS$ (and hence rule out an equilibrium with an attack)?