Simple FTPL, AM/PF

Christopher A. Sims
Princeton University
sims@princeton.edu

November 9, 2016

Continuous time flex-price FTPL model

Reviewing the model:

Household:

$$\max_{C,B,M} \int_0^\infty e^{-\beta t} \log C_t \, dt \quad \text{subject to}$$

$$C \cdot \left(1 + \gamma \frac{PC}{M}\right) + \frac{\dot{B} + \dot{M}}{P} \leq Y + \frac{rB}{P} - \tau$$

$$B \geq 0 \quad M \geq 0 \, .$$

Reorganized household FOC's

$$\text{intertemporal}: \quad r - \frac{\dot{P}}{P} - \beta = \frac{\dot{C}}{C} + \frac{2\gamma\dot{v}}{1+2\gamma v}$$

$$\text{liquidity preference}: \quad r = \gamma v^2$$

Government

$$\text{GBC}: \quad \dot{B} + \dot{M} + \tau P = rB$$

$$\text{Fiscal policy}: \quad \begin{cases} \text{active}: & \tau \equiv \bar{\tau} \\ \text{passive}: & \tau = -\phi_0 + \phi_1 \frac{B}{P} \end{cases}$$

$$\text{Monetary policy}: \quad \begin{cases} \text{active} & \dot{r} = \theta_0 \left(\theta_1 \frac{\dot{P}}{P} - r + \beta \right) \text{ or } M \equiv \bar{M} \\ \text{passive} & r \equiv \bar{r}\,. \end{cases}$$

In this set of slides we consider the AM/PF combination, with the $M \equiv \bar{M}$ version of "active money". (Another version of active monetary policy is $\theta_0 > 0$, $\theta_1 > 1$.)

Using the two reorganized FOC's to get an equation in v

We also are using the fact that with M constant

$$\frac{\dot{v}}{v} = \frac{\dot{P}}{P} + \frac{\dot{C}}{C}\,.$$

We arrive at

$$\gamma v^2 = \beta + \frac{\dot{v}}{v} + \frac{2\gamma\dot{v}}{1+2\gamma v}\,.$$

Rearranging,

$$\frac{1+4\gamma v}{v(1+2\gamma v)(\gamma v^2-\beta)}\dot{v} = 1\,.$$

This is an unstable differential equation in v with a unique stable solution $v \equiv \bar{v} = \sqrt{\beta/\gamma}$.

The nature of unstable solutions

If we integrate both sides of the equation from 0 to t, the right-hand side is just t and the left-hand side is

$$\int_{v_0}^{v_t} \frac{1 + 4\gamma v}{v(1 + 2\gamma v)(\gamma v^2 - \beta)}\, dv$$

If we began with $v_0 > \sqrt{\beta/\gamma}$, it is clear that v grows without bound, since $\dot{v}$ is always positive and, at least for large enough v, $\dot{v}/v$ steadily increases. But the left-hand side integral is bounded, no matter how large v_t becomes, as its integrand is $O(v^{-3})$ as $v \to \infty$. Since the right-hand integral's value of t is unbounded, this means that at some finite date t, v reaches infinity.

The nature of unstable solutions

If instead we begin with $v < \bar{v}$, $\dot{v}$ is negative and remains so. The integrand on the left hand side is negative, and in the limits of integration $v_0 > v_t$, so the integral is always positive. As $v \to 0$, the integrand is $O(1/v)$ and thus does not have a bounded integral, so v converges to zero rather than reaching zero in finite time. Also $\dot{v}/v \to -\beta$.

Thus starting from any $v < \bar{v}$, v decreases, eventually at the exponential rate $-\beta$. C converge to Y, and inflation $\dot{P}/P$ converges to $-\beta$.

The economics of the unstable solutions

- We suppose that the monetary policy must be implemented by a standard central bank, which maintains its constant-M policy through open market operations and turns over profits to the treasury.

- We assume (as does the usual government accounting system) that transfers from the central bank are treated as revenue by the treasury, and that the b in the fiscal policy rule does not include debt held by the central bank.

Fiscal policy and consumption on upward explosive paths

- In "cashless limit" models, and in many models including money, the real rate of return is either exactly β, the discount rate, or converges to it on any path satisfying the Euler equations.
- However in this model, because consumption is driven to zero along the upward explosive paths, the real rate of return does not converge to β.

Real return on bonds

$$r - \frac{\dot{P}}{P} = \gamma v^2 - \frac{\dot{P}}{P}$$

from SRC $\quad \dfrac{\dot{C}}{C} = \dfrac{-\gamma \dot{v}}{1 + \gamma v}$

from v definition $\quad \dfrac{\dot{P}}{P} = \dfrac{\dot{v}}{v} - \dfrac{\dot{C}}{C}$

$$\therefore \quad r - \frac{\dot{P}}{P} = \gamma v^2 - \frac{\gamma \dot{v}}{1 + \gamma v} - \frac{\dot{v}}{v}$$

from previous DE in v $\quad = \dfrac{\gamma^2 v^3}{(1 + \gamma v)(1 + 4\gamma v)} + O(1)\beta\,.$

Implications for real debt on explosive path

$$\dot{b} = \left(r - \frac{\dot{P}}{P}\right) b + \phi_0 - \phi_1 b$$

No matter how big ϕ_1 is, as the real return on debt rises, this equation eventually implies upward explosion in b at greater than an exponential rate.

$$\text{TVC:} \quad e^{-\beta t} \frac{B + M}{PC(1 + 2\gamma v)} \to 0\,.$$

$M/P \to 0$, but as we have seen B/P grows faster than exponentially.

$$C \cdot (1 + 2\gamma v) = \frac{Y(1 + 2\gamma v)}{1 + \gamma v}$$

remains bounded as $v \to \infty$, so the TVC is not satisfied.

Implied behavior for initial $v > \bar{v}$

- If initial P, and therefore v, is above the steady-state value, people who see the path implied by the Euler equations have an arbitrage opportunity: get rid of government bonds immediately.

- There is therefore an equilibrium with $v > \bar{v}$, but there is only one, and it is the equilibrium with valueless money.

- This depends on interpreting the fiscal rule with $P = \infty$, but since the rule is in real terms, it is natural to interpret it as implying debt becomes real debt in that case.

- Of course consumption is zero and utility $-\infty$ in the valueless-money equilibrium.

Further discussion of upward explosion

While this model does not have a continuum of upward-explosive equilibria, small variations on it do have this form of indeterminacy:

- Consider a trigger policy that kills inflationary paths without causing jumps in the price level, e.g. switching to $r \equiv \bar{r}$, $\tau = \bar{\tau}$, chosen so as to avoid a jump in P, when P or $\dot{P}/P$ hits some high critical value.

- Assume a transactions technology in which barter equilibrium is not such a disaster, e.g. with $C(1+\gamma v/(1+v))$ replacing $C(1+\gamma v)$ in the budget constraint. The next time I present this material I will concentrate on this transactions technology, as it's less unrealistic and not really harder to analyze.

- Assume fiscal policy is $\dot{B} = 0$.

Policy on downward paths

- Real balances go to infinity.

- A central bank must regularly roll over its assets, actively buying or selling to maintain constant M. Its holdings of real government debt therefore eventually grow without bound, at the exponential rate β.

- Since passive fiscal policy keeps b, debt in the hands of the public, on a stable path to a finite limit, The fraction of public debt held by the central bank (assuming its assets are all public debt) converges toward 100%.

- There are large offsetting flows of interest from the treasury to the central bank, and of profits from the central bank to the treasury.

- The steady deflation tends to increase b, so passive fiscal policy requires conventional surpluses ($\dot{B} < 0$).

Ruling out the deflationary solutions

- Looking back again at the transversality condition, we can see that with inflation at the rate $-\beta$, the transversality condition is *not* satisfied.

- Individuals find themselves becoming arbitrarily wealthy in real balances.

- An individual, taking the path of prices and interest rates as given, sees the effect on C of a decrease of δM in M as having an effect smaller than δC on consumption (from the budget constraint).

- As the price level drops, though, the value of δM in terms of consumption goods grows without bound. The utility value of "eating" a fraction δ of real balances today must therefore eventually exceed the discounted present value of the future losses of utility from increased transactions costs.

Plausibility of policy on the impossible deflationary paths

- The result that the TVC makes the deflationary paths impossible relies on the treasury stabilizing only the real value of the debt in the hands of the public, not counting the debt held by the central bank.
- Benhabib, Schmitt-Grohé, and Uribe (2001) work out a model in which AM/PF does allow deflationary equilibria, by assuming that fiscal policy stabilizes the real value of all debt, including that held by the central bank.
- The deflationary paths are eliminated by assuming that the public believes debt held on the central bank balance sheet is not backed by future taxes.

- If people believe that the rapid inflation required to return to the non-inflationary equilibrium price level would engender increased primary surpluses, the deflationary equilibria might be sustainable.

Policy to eliminate the upward-explosive equilibria

- The spiraling conventional deficits on these paths might (properly) be seen as a source of the high inflation.

- If high enough inflation produces a fiscal response, even if only a reduction in the conventional deficit below the value needed to keep real debt from declining, the explosive paths become unsustainable.

- The logic is simple. As soon as $v_0 > \bar{v}$, people foresee that inflation will be high and thus that primary surpluses will be higher than necessary to sustain the current real value of the debt.

- This increases demand for government paper, reduces the price level, and brings v back down to $\bar{v}$.

*

References

Benhabib, J., S. Schmitt-Grohé, and M. Uribe (2001): "The Perils of Taylor Rules," *Journal of Economic Theory*, 96, 40–69.