SIMS QUESTION FOR MACRO GENERAL

FTPL in a simple Keynesian model

A representative consumer maximizes

$$\int_0^\infty e^{-\beta t} \log C_t \, dt \quad \text{subject to}$$

$$C_t + \tau + \frac{\dot{B}_t}{P_t} = Y_t + \frac{rB_t}{P_t}\,.$$

There is no uncertainty after the initial date. The consumer chooses the time paths of C and B, taking r, P, Y_t and τ as given.

Government policy fixes r and τ at positive, constant values. To keep the algebra simple, we assume $r = \beta$. Since this is a Keynesian model, instead of an endowment process or a production function we introduce a Phillips Curve, here an old-fashioned backward-looking one:

$$\dot{p} = \gamma(y - \bar{y})\,,$$

where p is the log of the price level and $\bar{y}$ is a normal, or full-employment, level of the log of output. Again to keep the algebra simple, we assume $\bar{y} = 0$.

The government budget constraint is

$$\dot{B} = rB - P\tau\,.$$

Note that the Phillips curve is not forward-looking, so it implies that p (and thus also P) cannot jump discontinuously at the initial date. Also, the government budget constraint implies that B cannot jump at the initial date.

(a) Display the social resource constraint.

Dividing the government budget constraint by P and subtracting it fromthe private budget constraint delivers $C_t = Y_t$.

(b) Using the private agent's optimality conditions and the other equations of this model, derive a differential equation system in real debt b and and the log of consumption c that must be satisfied in equilibrium.

The FOC's are

$$\partial C: \qquad \frac{1}{C} = \lambda$$

$$\partial B: \qquad \frac{-\dot{\lambda}}{P} + \frac{\lambda}{P}\frac{\dot{P}}{P} + \beta\frac{\lambda}{P} = r\frac{\lambda}{P}\,.$$

Solving to eliminate λ gives us

$$\frac{\dot{C}}{C} = r - \beta - \frac{\dot{P}}{P}\,.$$

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Using the social resource constraint and the Phillips curve, and using lower case c for the log of consumption, this gives us

$$\dot{c} = r - \beta - \gamma(c - \bar{y}) . \qquad (*)$$

This equation is forward-looking, as it is based on the B FOC, so it does not rule out initial jumps in c. But it determines the time path of c from any given initial condition.

The government budget constraint becomes

$$\dot{b} + b\dot{p} = rb - \tau = \dot{b} + b\gamma(c - \bar{y})$$

(c) Linearizing if necessary, determine whether the model has any stable solution and if so, whether it is unique.

The model has a steady state, where $r = \beta$, $\dot{c} = 0$, and hence $c = \bar{y}$ and $\dot{p} = 0$. The private FOC equation $(*)$ is already linear. A steady state for b therefore requires

$$b = \bar{b} = \tau / r$$

. Then linearizing the GBC delivers

$$\dot{b} + \bar{b}\gamma(c - \bar{y}) = rb - \tau ,$$

The differential equation $(*)$ above is stable and implies (with $r = \beta$)

$$c_t = \bar{y} + (c_0 - \bar{y})e^{-\gamma t} .$$

Solving the GBC forward gives us

$$b_0 = \int_0^\infty e^{-rt}(\tau + \bar{b}\gamma(c - \bar{y}))\, dt = \frac{\tau}{r} + \frac{\bar{b}\gamma(c_0 - \bar{y})}{\gamma + r} . \qquad (\dagger)$$

Since both B_0 and P_0 can't jump, b_0 is predetermined, so c_0 must adjust to make the equation above hold. Initial $\dot{p}$ is then determined by the Phillips curve and $\dot{c}$ by $(*)$. All initial values are thus uniquely determined.

(d) Determine how initial c and $\dot{p}$ move if there is a one-time, unanticipated increase in τ, with the economy initially in steady state.

Equation $(\dagger)$ involves only future τ values, so it holds with the new higher value of τ in place. But then it is easy to see that, since b_0 is fixed, c_0 must decline to offset the increase in τ. The fall in c_0 lowers the inflation rate, thereby increasing the real rate of interest, thereby discounting the larger stream of future τ's more heavily to make their discounted value still match b_0. It is also possible to derive this conclusion without linearizing the government budget constraint, instead solving it forward in its nonlinear form and differentiating with respect to c and τ at the $c = \bar{y}$ steady state.