Consider a monopolist who must each period set a price. The demand curve she faces is $Q = 1 - P$, where Q is quantity sold and P is the price. Her unit cost is X, a random variable with a $N(\mu, \sigma^2)$ distribution. The monopolist maximizes expected profits, less an information cost measured by Shannon mutual information between P and X. Formally, the problem is

$$\max_{f(p,x)} E[(P - X)(1 - P)] - \theta I(P, X)$$

$$\text{subject to} \quad \int f(p,x)\, dp = \phi(x; \mu, \sigma^2)\,, f(p,x) > 0\,,$$

where $I(\cdot,\cdot)$ is mutual information, f is the joint density of X and P, and $\phi()$ is the normal density function.

We are assuming implicitly here that the solution has the form of a density with respect to Lebesgue measure. Note also that we are allowing some probability of negative costs, and we are not constraining Q or P to be positive. If instead we had constrained prices and quantities to be non-negative, the optimal information-constrained behavior for the monopolist would not be solvable analytically and would involve discrete distributions for price.

Certainty equivalence applies here, so that the solution will make the optimal P value that which solves the non-stochastic version of the problem with $\hat{x} = E[X \mid P]$ replacing X.

The optimal solution does make X and P jointly normal, but this is not a pure tracking problem — losses are not a function of $\|p^* - p\|$ (where p^* is the unconstrained solution) alone. By using the certainty-equivalence value of the conditional mean of $X \mid P$, plus the knowledge that the solution is jointly normal, plus the FOC's of the problem (ignoring the $f(x,y) > 0$ constraints, because these do not bind in the solution, except when the solution collapses to a single point), find:

(1) how the conditional variance of $X \mid P$ behaves as a function of θ and the range of values of θ over which the solution gives the distribution of P full support on the real line;

(2) how $E[P \mid X]$ behaves as a function of X.