POLICY TO EXIT A LIQUIDITY TRAP

Consider this stripped-down, flexible-price, no-money fiscal theory model. The representative agent solves

$$\max_{C,B} \int_0^\infty e^{-\beta t} \log(C_t) \, dt \quad \text{subject to} \tag{1}$$

$$C + \frac{\dot{B}}{P} = \frac{rB}{P} - \tau + Y \tag{2}$$

$$B \ge 0 \tag{3}$$

where *C* is consumption, *B* is nominal government bonds, *P* is the price level, $p = \log(P)$, *r* is the nominal interest rate, τ is lump-sum taxes, and *Y* is an endowment stream. We assume *Y* constant. There is no randomness, and we consider only perfect foresight solution paths. You will need to use the fact that private agent transversality rules out permanent upward growth in total wealth (real bond holdings, plus the discounted present value of future endowment, minus the discounted present value of future taxes) at a rate of β or greater. This is a fully specified equilibrium model. Non-stationary paths can be ruled out only by showing they are infeasible or violate a transversality condition.

Government has the budget constraint

$$\frac{\dot{B}}{P} = \frac{rB}{P} - \tau , \qquad (4)$$

and must set two of the policy variables r, B and τ , or else two distinct functional relations connecting them to each other or to other variables — usually one interpretable as "monetary policy" and one as "fiscal policy". We assume that at time t = 0 the government has inherited from past bad policy a situation in which r has been constant at zero for some time. This situation arose from a policy combination that set monetary policy and fiscal policy as

$$\dot{r} = \begin{cases} \max(\theta_r \cdot (\theta_\pi (\dot{p} - \pi^*) - r + \beta + \pi^*), 0) & \text{if } r = 0\\ \theta_r \cdot (\theta_\pi (\dot{p} - \pi^*) - r + \beta + \pi^*) & \text{if } r > 0 \,. \end{cases}$$
(5)

$$\tau = -\phi_0 + \phi_1 \frac{B}{P} \,, \tag{6}$$

with $\phi_0 > 0$, $\phi_1 > \beta$, $\theta_\pi > 1$, $\theta_r > 0$. The original version of the exercise introduced *b* as notation for real debt in the fiscal policy rule, defining it only below.

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FOC's for the private agent are

$$\partial C: \qquad \frac{1}{C} = \lambda$$

$$\partial B: \qquad -\frac{\dot{\lambda}}{P} + \frac{\beta\lambda}{P} + \frac{\lambda}{P}\frac{\dot{P}}{P} = \frac{r\lambda}{P},$$

which reduce to

$$\frac{\dot{C}}{C} = r - \frac{\dot{P}}{P} - \beta \,.$$

This equation holds along the the equilibrium path after the initial date and does not have implications for whether *C* or *P* or any combination of them has continuous paths. The government budget constraint and the monetary policy equation are not forward-looking, though, so they imply that *B* has continuous time paths and that $r - \theta_r \theta_\pi p$ has continuous time paths whenever r > 0. Note that this allows for the possibility of a jump from r = 0 to a positive level of *r* at the initial date if *p* jumps upward at that date.

The social resource constraint is C = Y, so C is constant, which implies

$$\frac{\dot{P}}{P}=r-\beta\,.$$

(a) Show that if the government sticks with these policies the zero interest rate can persist forever. How will the price level and real government debt b = B/P behave in this case?

r fixed at 0 implies that $\dot{P}/P = -\beta$, so there will be persistent deflation. The fiscal rule is passive and together with government budget constraint and the agent FOC implies that, whatever the initial level of real debt, real debt will converge exponentially to a steady state value of $\phi_0/(\phi_1 - \beta)$. Since nothing else pins it down, initial *P* is indeterminate. If it jumps up at all, though, it will make *r* jump up above zero. (In effect, the inflation rate becomes infinite temporarily, pushing *r* up instantly.) This will get the economy out of the zero-*r* equilibrium, and if the time-zero indeterminacy is the only "surprise" allowed for, the economy will stay in the new equilibrium with r > 0. But if the price stays constant or jumps downward, policy will continue to imply r = 0 and the equilibrium with zero interest rate can persist indefinitely.

- (b) Show that with these policies the initial price level is indeterminate and can jump discontinuously at time zero. (Recall that Euler equation first-order conditions in this kind of model restrict only right-derivatives and that a monetary policy rule like that specified here allows upward jumps at time zero in *p* so long as $\Delta r = \Delta p$.) Is it possible that an initial jump in the price level could take the economy out of the liquidity trap? Permanently? Temporarily? Assume here that jumps are only at time zero, with no later "sunspot" jumps. See above.
- (c) Suppose the monetary authority announces that, rather than increasing *r* as soon as its policy rule above implies $\dot{r} > 0$, it will keep $r \equiv 0$ until $\dot{p} = \pi^* > 0$, and

only then start (permanently) using the policy rule (5). Fiscal policy would remain unchanged. Would this guarantee exit from the liquidity trap? Why or why not?

Since we have already verified that an equilibrium with r = 0 and deflation forever is possible, a commitment to keep r at zero until inflation hits some positive target will do nothing to alter expectations of future policy in that equilibrium.

(d) Suppose the fiscal authority announces that it will cut the primary surplus τ to a new low level, not anticipated by the public before time 0, and that it will keep the primary surplus at this low level until $\dot{p} = \pi^*$, after which it will revert permanently to (6). Monetary policy continues to follow (5) Will this end the liquidity trap? The price level indeterminacy? Why or why not?

Suppose agents think that despite this new fiscal policy, deflation will persist forever. Then they think that the primary surplus τ will stay at its new low level forever. This is active fiscal policy, and will require $B/P = \tau/\beta$ to avoid violating transversality. (We assume that B < -0, borrowing from the government is impossible, or at least bounded.) If the new level of τ is low enough, it will require an upward jump in P, which will push r above zero and violate the assumption that agents think r will be zero forever. So this would eliminate the permanent ZLB equilibrium. However, if initial real debt b is high enough, the passive fiscal policy implies a steadily declining τ , so that the new τ could be below that implied by the old policy, yet still imply a higher discounted present value of primary surpluses. This would require a downward jump in P, which can occur without requiring any movement of r away from zero. So announcing a small decrease in τ , to be maintained until inflation hits a positive target, might leave the ZLB equilibrium undisturbed, while eliminating the price level indeterminacy (because now fiscal policy would be permanently active).

If prices initially jump upward the inflation rate is temporarily infinite, which we can interpret as implying an immediate reversion to passive fiscal policy. Since the passive fiscal rule makes τ decrease when *b* decreases, τ will jump downward, though possibly not by the same amount announced as the level of τ to be implemented so long as *r* stayed at zero. Price level indeterminacy would remain, since any upward jump in *P* ends the ZLB equilibrium, but the policy would end the ZLB equilibrium.

If we interpret the policy commitment to switch back to passive fiscal policy as applying only after the initial date, things get more complicated. If the initial upward jump in rand p is small, we would still have $r < \beta$ and thus deflation, just at a less negative rate than at the ZLB. Then the Taylor rule would still imply $\dot{r} < 0$. Then, if the commitment to return to passive fiscal policy is never invoked, the price level would be determinate (because fiscal policy would be active) and r would follow a path back to the ZLB. If the initial jump in r pushes $\dot{p} = r - \beta$ above target right away, we are back in the case where fiscal policy reverts to passivity immediately. If the initial jump in r is not that large, but still large enough to put r on an upward-explosive instead of downwardexplosive path, the switch to passivity will occur in finite time. Since whenever it occurs, and whatever real debt level prevails at that point, the passive fiscal policy guarantees satisfaction of transversality, any initial price level in this range is possible. (e) Suppose the government announces that it will switch permanently to a policy that fixes the growth rate of nominal debt to be constant $\dot{B}/B = \gamma$. Will this end the liquidity trap? The price level indeterminacy? Why or why not?

The ZLB equilibrium has P shrinking at the fixed rate $-\beta$. If γ is larger than this, the commitment to a fixed growth rate for B is inconsistent with the ZLB equilibrium. The Taylor rule, as usual, implies that there is a unique initial price level consistent with non-explosive behavior of r and inflation, and that price level implies constant r and constant inflation. If that inflation rate is greater than $\gamma - \beta$, b grows exponentially at less than the rate β , so transversality is satisfied. So in this case there is an equilibrium with non-explosive inflation and that equilibrium is associated with a unique initial price level. However, at initial price levels above the one associated with stable inflation, inflation explodes upward, and these paths satisfy all the conditions for equilibrium. Real debt eventually shrinks toward zero on these paths, because eventually $\gamma - \dot{p}$ becomes negative, and this is true regardless of the initial level of B.

(f) Suppose the government announces convincingly that it has identified a portion \overline{B} of the current nominal debt as irresponsibly generated by a previous administration and that it will therefore not subject the public to taxes to back this debt. The fiscal rule is therefore

$$\tau = -\phi_0 + \phi_1 \frac{B - \bar{B}}{P} \tag{7}$$

instead of (6) above, still with $\phi_0 > 0$, $\phi_1 > \beta$. \overline{B} , the ignored part of the debt, is constant in nominal value. Does this end the liquidity trap? The indeterminacy?

With this fiscal policy and a persistent ZLB, the government budget constraint becomes

$$\dot{b}=eta
ho+\phi_0-\phi_1b+\phi_1ar{b}e^{eta t}$$
 ,

where $\bar{b} = \bar{B}/P_0$. The steady deflation at rate $-\beta$ at the ZLB produces the last term in this equation. This equation implies

$$b_{t} = \int_{0}^{t} e^{(-\phi_{1}+\beta)s} \left(\phi_{0} + \bar{b}e^{\beta(t-s)}\right) ds + b_{0}e^{(-\phi_{1}+\beta)t}, \text{ which implies}$$

$$b_{t} = \phi_{0} \frac{1 - \exp((-\phi_{1}+\beta)t)}{\phi_{1} - \beta} + b_{0}e^{(-\phi_{1}+\beta)t} + \bar{b}e^{\beta t} \frac{1 - \exp(-\phi_{1}t)}{\phi_{1}}.$$

The last term in this expression is positive, and grows at the rate β , which violates transversality. This policy therefore is inconsistent with a ZLB equilibrium. The price level therefore must jump up at the initiation of this policy, pushing *r* above zero. The jump must be large enough so that the economy does not come back to the ZLB equilibrium. If it is exactly to the value consistent with constant inflation under the Taylor rule, and this level of inflation is positive, then the solution for the time path of real debt is non-explosive and transversality is satisfied. This is true also if the jump in price is larger, so that the economy is in an equilibrium with upwardly explosive

inflation. So this policy eliminatetes the ZLB equilibrium, but leaves the price level indeterminate.

(g) What about a fiscal rule that adds to the right hand side of (7) a term $\phi_2 \dot{p}$. Does this end the liquidity trap? The indeterminacy?

Since in the ZLB equilibrium \dot{p} is constant at $-\beta$, the new term just adds a negative constant to the right-hand side of the fiscal rule so as in the previous case real debt would explode at the rate β violating transversality. The equilibrium with constant positive inflation is still an equilibrium, by the same argument as before. Now, though, equilibria with upwardly explosive inflation are impossible, if they imply inflation growing at a rate greater than β . This is true on any unstable path for \dot{p} so long as $\theta_r(\theta_{\pi} - 1) > \beta$, but not otherwise. So the policy eliminates the ZLB and may eliminate indeterminacy also, if monetary policy responds strongly and quickly enough to inflation.

(h) Finally, what about simply shifting to the simplest active fiscal, passive money configuration, with the policy behavior now $r \equiv \bar{r}$ and $\tau \equiv \bar{\tau}$ instead of (5)-(6). This can certainly end the liquidity trap and the non-uniqueness. In this model, with zero-duration debt, can it be done in such a way that the price level does not jump at the initial date, even though *r* must jump? How do \bar{r} and $\bar{\tau}$ have to be set to achieve this?

B can't jump and we want to prevent a jump in *P*, so we take initial *P* as given. The new equilibrium has $B/P = \bar{\tau}/\beta$. So to prevent the *P* jump, we must set $\bar{\tau}$ so this equation is satisfied with the given values of *B* and *P*. If we do so, it doesn't matter how we set \bar{r} . That affects only the inflation rate in the new equilibrium.