MACRO GENERAL EXAM

(1) Consider the simple flexible price equilibrium model of an endowment economy with government debt. Individuals maximize, choosing time paths for C and B,

$$E\left[\sum_{t=0}^{\infty} \beta^t \log C_t\right]$$

subject to

$$C_t + \frac{B_t}{P_t} = R_{t-1}\frac{B_t}{P_t} - \tau_t + Y_t\,.$$

The government sets the gross nominal interest rate R and the primary surplus τ.

For each of the following possible combinations of interest rate and primary surplus policies, discuss whether an equilibrium exists and whether, if it exists, it is unique. Also determine whether equilibria that exist have stable inflation. Note that there is no money in this model, so negative interest rates are possible. Assume Y_t is constant. Assume the government cannot lend to the public (i.e., $B \geq 0$).

There was a typo in the budget constraint: the B_t on the right-hand-side should have been B_{t-1}. People seemed either not to notice it, writing out the budget constraint properly without comment, or to just realize it was a typo and say so.

The government budget constraint was not specified. You were expected to realize it is

$$\frac{B_t}{P_t} = R_{t-1}\frac{B_{t-1}}{P_t} - \tau_t\,, \quad \text{implying the SRC}$$
$$C_t = Y_t$$

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Private agent FOC's are

$$\partial C: \qquad \frac{1}{C_t} = \lambda_t$$

$$\partial B: \qquad \frac{\lambda_t}{P_t} = R_t \beta E_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right] \tag{$**$}$$

Because Y_t is specified as constant, there is no fundamental source of uncertainty in this model, and some people dropped the expectation operator in the B FOC, on the grounds that the solution would be deterministic. This was OK, since it didn't really change the analysis, but in cases of indeterminacy, the solution can be stochastic despite the absence of any fundamental uncertainty. (I.e., "sunspot equilibria" are possible.)

Solving to eliminate the Lagrange multiplier gives us

$$R_t \beta E_t\left[\frac{P_t}{P_{t+1}}\right] \tag{$*$}$$

(a)

$$R_t \equiv \bar{R}\,, \quad \bar{R} = \beta^{-1} \tag{1}$$

$$\tau_t \equiv \bar{\tau}\,, \quad \bar{\tau} > 0 \tag{2}$$

This is the classic active fiscal, passive money case, and produces a unique price level. The quick and dirty version of the argument is that, with this policy configuration, we can solve the government budget constraint forward to obtain

$$b_t = \frac{B_t}{P_t} = \frac{\bar{\tau}}{\beta^{-1} - 1} \equiv \bar{b}\,.$$

But then the one-period budget constraint at time zero is

$$\bar{b} = R_{-1}\frac{B_{-1}}{P_0} - \bar{\tau}\,,$$

and everything in this equation other than P_0 is either given as an initial condition (the variables dated -1) or fixed by the forward solution of the GBC ($\bar{b}$). so we can use the equation to solve for P_0, assuming $B_{-1}R_{-1} > 0$.

A better answer would observe that the forward solution for b_t is derived assuming

$$E[\beta^t b_t] \xrightarrow[t\to\infty]{} 0\,.$$

That this can't have a negative lim sup follows from the $B_t \geq 0$ constraint. That it can't have a positive lim sup looks like a transversality condition, but isn't quite in this case. The transversality condition of the private agent here is

$$\beta^t E\left[\frac{b_t}{C_t}\right] \to 0.$$

However, since $C_t \equiv Y$, this transversality condition implies the condition needed to derive the forward solution of the budget constraint.

(b)

$$R_t = \left(\frac{P_t}{P_{t-1}}\right)^\theta \beta^{-1}\,, \quad \theta > 1 \tag{3}$$

$$\tau_t \equiv \bar{\tau}\,, \quad \bar{\tau} > 0 \tag{4}$$

This is an active monetary policy combined with active fiscal policy. We should expect there to be no stable solution except in knife-edge special cases. But there might be an unstable solution that is an equilibrium.

With τ fixed, the same reasoning as in the first part leads to

$$b_t = \bar{b} \equiv \frac{\bar{\tau}}{\beta^{-1} - 1}\,,$$

and as before we can derive from this a unique value for P_0. The Taylor rule for R then gives us R_0. B_0 is determined by $B_0/P_0 = \bar{b}$. At $t = 1$, with R_0 and B_0 given, we can derive a unique P_1, this in turn gives us B_1, R_1, etc. Since at each step of this forward recursion we are using the Taylor rule and the real debt evaluation equation, we can be sure these two equations are satisfied along the equilibrium path. But do we satisfy the FOC $(*)$? The explicit solution for P_t each period is

$$P_t = \frac{R_{t-1}B_{t-1}}{\bar{b} + \bar{\tau}}\,.$$

and using $B_{t-1} = \bar{b}P_{t-1}$, we can verify $(*)$. Note that $(*)$ only holds for $t \geq 0$. Also that there is no sunspot randomness. The time path of P_t is uniquely determined.

However, substituting $R_t = \beta^{-1}P_{t+1}/P_t$ into the monetary policy equation, we get

$$\beta^{-1}\frac{P_{t+1}}{P_t} = \beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\theta}. \qquad (\dagger)$$

With $\theta > 1$ this implies that gross inflation grows at an exponential rate if it is initially above one and shrinks toward a gross rate of zero if it is initially below one. Only if P_0/P_{-1} turns out to be one will this equation be consistent with a stable path of prices, but we have found P_0 from $R_{-1}B_{-1}$; P_{-1} could therefore have any value without changing P_0. The solution will therefore generally give an explosive price path.

(c)

$$R_t = \left(\frac{P_t}{P_{t-1}}\right)^{\theta}\beta^{-1}, \quad \theta > 1 \qquad (5)$$

$$\tau_t = -\phi_0 + \phi_1\frac{B_{t-1}}{P_{t-1}}, \quad \phi_1 > \beta^{-1} - 1 \qquad (6)$$

This is an active money, passive fiscal specification. We expect there is only one stable solution, but is it unique?

The fiscal rule guarantees that real debt is on a stable path from any initial level. It cannot be solved forward to pin down the price level. With no sunspots, we could derive (†) as in the previous problem. It would give us a unique price level path from any value of P_0, but could not fix P_0. Thus any initial P_0 is possible, with only one giving a stable path. Because real debt is stationary on these paths, no transversality condition (or any other FOC) is being violated.

A more careful answer would allow for sunspots, so that the E_t operator in ($*$) has to be retained. Then instead of (†), we get

$$E_t\left[\frac{P_t}{P_{t+1}}\right] = \left(\frac{P_{t-1}}{P_t}\right)^{\theta}$$

Because of the expectation operator, this does not deliver a unique path for any given initial P_0. In other words, sunspot equilibria, in which the inflation rate responds to non-fundamental sources of randomness, are possible. The solutions must be unstable, in the sense that if initial P_0/P_{-1} exceeds one, the *expected* time path of prices explodes upward at an ever increasing rate, and vice versa if it starts below one.

(d) It can be argued from history that government fiscal restriction is not motivated by high debt, but only by high interest expense in the budget, as conventionally measured. So consider this policy:

$$R_t = \left(\frac{P_t}{P_{t-1}}\right)^{\theta} \beta^{-1}, \quad \theta > 1 \tag{7}$$

$$\tau_t = -\phi_0 + \phi_1 (R_{t-1} - 1) \frac{B_{t-1}}{P_t} \tag{8}$$

Since we again have a Taylor rule with $\theta > 1$, we know that limits us to a stable path with prices constant, or paths with gross inflation exploding toward infinity or zero. Are any or all of these consistent with transversality, $B_t \geq 0$, and the GBC?

Using the B FOC $(**)$ in the government budget constraint we get, along any equilibrium path,

$$E_t b_{t+1} = \left(\beta^{-1}(1 - \phi_1) + \phi_1\right) b_t + \phi_0 \,.$$

For any $\beta < 1$ and $1/(1 - \beta) > \phi > 0$ this is a slowly upward-explosive difference equation in b, growing at a rate less than β^{-1}. Such growth does not violate transversality. In fact here, because lump sum taxes eventually grow at the same rate as b, private agents will not even see it as feasible to reduce b, since without it they would not have resources to pay the future taxes they see as exogenously given.

But this means that real debt does not pin down an initial P_0 here, so the price level is indeterminate. Of course upward explosive paths imply that the lump sum taxes grow without bound, which is not realistic, and the downward explosive paths require that lump sum *transfers* grow without limit, which is also not realistic.