COMMENT ON THE RATIONAL INATTENTION EXERCISE

Several students noted that the FOC's for an optimum in the problem are apparently all satisfied when there is a single point of support for the distribution of prices, i.e. in the no-information solution, regardless of the cost of information. Though the objective function is not globally concave in price, it is locally concave where the FOC's are satisfied, so the profit-maximizing price is unique and solves the FOC's. The objective function is concave and the constraints convex in the probabilities, so solutions to the FOC's should always define an optimum. However, there are Kuhn-Tucker first order conditions corresponding to the $p_{ij} \geq 0$ constraints at every value of prices that has zero probability. One can put these extra FOC's in simple form by writing the probabilities as $p(\pi)q(c \mid \pi)$, where p is the marginal density for π and $q()$ is the conditional density for costs given price. Then the FOC w.r.t. p where $p = 0$ is

$$\int \big(U(\pi, c) - \alpha \log(q(c \mid \pi)) - \mu(c)\big) q(c \mid \pi)\, dc \leq 0\,,$$

where α is the cost of information and $\mu(c)$ is the Lagrange multiplier on the constraint that imposes the marginal density for c. This must hold for any choice of q. When α is low enough, this condition will fail for some values of π in the no-info solution.

This fact creates a potential problem in numerical computation. Iterating on the FOC's for a three-point solution to reach a fixed point, which is what everyone who successfully solved the problem did, can get stuck at a solution with too few points of support. If one tries to solve for a three-point-of-support solution when only two are needed, it is possible (only a couple of people succeeded in this, though) to structure the fixed point algorithm so it still behaves nicely, converging to a solution in which the values of π and the conditional distribution for $c \mid \pi$ are the same for two or more points in the solution. However, it is also possible for such an algrithm to get stuck at a solution with two points of support or one point of support, since the FOC's that are actually being used are all satisfied there, even though a better solution is possible with more points of support.