EXERCISE DUE 2/29/11: DIFFERENCE OF OPINION AND CAPITAL ALLOCATION

The economy has two types of agents, $i \in \{a, b\}$, and exists for two periods, $t \in \{1, 2\}$. Each agent solves

$$\max_{C_{1i}, C_{2i}, S_i, b_i} U(C_{i1}) + E[U(C_{i2})] \quad \text{subject to} \tag{1}$$

$$C_{i1} + S_i = Y \tag{2}$$

$$C_{i2} = RS_i + h_i(Z - q) + \pi\,, \tag{3}$$

where U is the period utility function, C is consumption, R is the rental rate for capital, h is the amount bet, q is the price of the bet, Z is a random variable (the thing being bet on), and π is the dividend from the firm. The production function makes output equal to $A \cdot (S_a + S_b)^\alpha$. The representative firm is a competitive profit-maximizer with no dynamics to its decisions, so its behavior guarantees $R = \alpha A \cdot (S_a + S_b)^{\alpha - 1}$ in equilibrium. Market clearing requires

$$C_{a2} + C_{b2} = A \cdot (S_a + S_b)^\alpha\,. \tag{4}$$

Also, bets are on a random variable Z that has no connection to the production technology, and bets are all among agents in the economy, so $h_a + h_b = 0$.

Both agents have the same utility function, $U(C) = -\exp(-\gamma C)$, which is the constant absolute risk aversion (CARA) form. They differ only in that they have different beliefs about Z. Agent i believes that $Z \sim N(\mu_i, 1)$. In other words, they differ in their beliefs about the mean value of Z.

Solve the model and determine whether there are analogues to the results described in class for this model with CRRA utility: i) when both agents have the same distribution for the random variable Z, there is no betting and thus no effect on allocation of the existence of Z; ii) when the distributions differ, betting occurs and each agent's second-period consumption is more uncertain than in the model without betting; and iii) with high risk aversion betting reduces total investment, and with low risk aversion betting increases total investment, compared to the model without betting.

The third result obviously can hold only in some modified form, since in the CRRA model the single *relative* risk aversion parameter controlled the result, while here it is absolute risk aversion γ that will be fixed across equilibria you compare. Results may then differ when wealth levels, here indexed by A, change.

Note that in contrast with CRRA utility, which is undefined for negative C, CARA utility is defined for negative C and makes marginal utility finite at $C = 0$. Sometimes modelers constrain C to be positive with CARA utility nonetheless, but to make solution easier here, assume that C can be negative. Since the agents believe

Z is normally distributed, they believe it could be negative and arbitrarily large in absolute value with positive probability, so if they bet at all they must see $C_2 < 0$ as feasible.

So that results are comparable across people in the class, Everyone should consider at least a base case with $A = 2$, $\alpha = .5$, $\mu_1 = \mu_2 = 1$, and a second one in which $\mu_1 = 1$, $\mu_2 = 2$.