

## FINAL EXAM FOR PART 1 OF THE COURSE

*You have two hours — 120 minutes — for the exam. Answer all questions. Points for each question are listed beside the question number, and the points add up to 120.*

- (1) (30 pt) Suppose a decision variable  $c$  can take on only the three values 1, 2, and 4, while the information variable  $w$  can take on only the three values 2, 4, and 8. The payoffs from the possible combinations of  $c$  and  $w$  values are given by this table:

	w		
c	2	4	8
1	0	1.6	2.8
2	$-\infty$	2	3.6
4	$-\infty$	$-\infty$	4

Before any observations are taken, the distribution of  $w$  puts probabilities .5, .25, and .25 on the points 2, 4, 8, respectively.

- (a) If there were no information costs to reducing uncertainty about  $w$ , what would be the optimal rule for choosing  $c$ ?  
 Set  $c = 1$  for  $w = 2$ ,  $c = 2$  for  $w = 4$ , and  $c = 4$  for  $w = 8$ .
- (b) What would be the mutual information, in bits, between  $c$  and  $w$  in this unconstrained solution?  
 Ex post the uncertainty about  $w$  would be resolved, so the entropy would be zero, and ex ante it is the entropy of the unconditional distribution, which is  $.5 \log_2 2 + .25 \log_2 4 + .25 \log_2 4 = 1.5$  bits, and this is the amount of mutual information.
- (c) What would be the optimal rule for setting  $c$  if it were impossible to collect any new information about  $w$ , so the mutual information between  $w$  and  $c$  must be zero?  
 Only  $c \equiv 1$  avoids putting positive probability on infinite losses, so all probability must concentrate on that point if  $c$  and  $w$  are independent (which is implied by zero mutual information).
- (d) Suppose the joint distribution of  $c$  and  $w$  is given by the table of probabilities below. Calculate the mutual information, in bits, between  $c$  and  $w$  for this case.

c	w		
	2	4	8
1	.5	.125	.125
2	0	.125	.125
4	0	0	0

We need to form: (entropy of marginal for  $c$ ) + (entropy for marginal of  $w$ ) - (entropy of joint distribution). This is

$$-(.75 \log_2 .75 + .25 \log_2 .25) + 1.5 + (-.5 \log_2 + 4 \cdot .125 \log_2 .125) \doteq .3$$

- (e) With the probabilities in the table, is the flow of mutual information less than in the case of the unconstrained solution? Is expected payoff higher than in the zero-information-flow case? Explain your answers.

The information flow is lower than in the unconstrained solution: .3 vs. 1.5 bits. We know in any case that it can't be any higher. The payoff is lower than the unconstrained case: expected utility of  $.5 \cdot 0 + .125 \cdot (1.6 + 2.8 + 3.6 + 2) = 1.25$  vs 1.5. You weren't asked about this.) The payoff is higher than in the zero-information flow optimum, because it puts the same probability on (1,2), while moving probability from (1,4) to (2,4) and from (1,8) to (2,8), which in each case increases expected utility. The expected utility for the table is 1.25, while for the unconstrained case it is 1.5 and for the zero-information-flow case it is 1.1.

Useful facts: Entropy in bits of  $X$  with pdf  $p()$  is  $H(X) = -E[\log_2(p(X))]$ . Mutual information between  $X$  and  $Y$  is the sum of the entropies of their marginals minus the entropy of their joint pdf.  $\log_2(3) = 1.6$ , approximately.

Answers to the remaining questions are not unique. These are samples.

- (2) (30 pt) Which of the three studies — Sargent-Williams-Zha, Sims-Zha, and Primiceri — implies the most optimistic conclusion about the likelihood of recurrence of an inflationary episode like the 70's in the US? In this first part of the answer take each study on its own terms; that is, here you are comparing the claims of the studies, not assessing which is correct. Then answer this question: Based on your own judgment about the strengths and weaknesses of these studies, is it likely that inflationary episodes like the 70's will regularly recur? Explain your answers, keeping in mind that your answer should show your understanding of the papers.

SWZ conclude there is essentially no conflict in the long run between commitment and no-commitment solutions to their model, and that learning will therefore take the policy authority very close to the Ramsey optimum. However, when they simulate their fitted model, in every case

(even when they simulate conditional on within-sample initial conditions) the simulations show widely oscillating inflation for decades. Thus the estimates imply that a repeat of the late 70's is likely, soon, indeed several such repeats are likely before the economy settles down. Sims-Zha find little evidence of any shifts in monetary policy and that existing monetary policy implies a stable path for inflation. However, their estimates include regimes in which the shocks to the private sector can generate (as they did in the late 70's) a rapid runup inflation, which is resolved only over time. Their estimates, when they insist on changing coefficients in the policy rule despite poorer fit, show that there is a "regime" that occurred regularly before 1979 that has occurred very rarely since. This could be read as suggesting that policy has improved over time. However their counterfactual simulations suggest that the recent policy rule would not have greatly reduced the amount of inflation had it prevailed in the 70's. Primiceri finds the policy makers learning over time and arriving by the end of the sample at close to the correct model. He, like SWZ, finds that the economy converges to a steady state with low inflation. His paper does not show the long period of wide swings in inflation in out of sample simulations that show up in the SWZ model, and in this sense is more optimistic than SWZ, and therefore the most optimistic of the three overall.

Strengths and weaknesses: Here there's definitely no one right answer. However the strength of SWZ and Primiceri is that they have complete behavioral stories for how the economy works, while SZ model explicitly only monetary policy. SZ have a less restrictive model that carefully models time varying variances and therefore probably fits better than the other two models. SWZ's model produces the anomalous out of sample projections and implies bizarre initial beliefs on the part of the policy makers, which might make one mistrust its structural story. Primiceri uses an "old-fashioned" Phillips curve with unemployment on the right and models the policy authorities as controlling unemployment directly, inflation only indirectly through the Phillips curve. This is the kind of model the policy makers probably had in mind during the 70's, but it fits uncomfortably with a rational expectations perspective. The SWZ model incorporates RE explicitly in its "true" model, but one has to balance this against the implausible parameter estimates that emerge from the model.

- (3) (10 pt) Explain why some economists argue that one can't properly measure a central bank's net worth without knowing what its current and future monetary policy is.

For a firm, net worth includes intangible assets — brand names, technical know-how accrued from experience, etc, — that will contribute to future revenue but cannot be marketed directly. A central bank can “earn” in the future by generating seignorage. One might therefore conclude that its net worth has to depend on the extent to which it plans to create future seignorage revenue, which depends on its monetary policy.

- (4) (20 pt) “A central bank that can print money can always pay its bills, so it can never go bankrupt. A central bank that has to be concerned about its balance sheet can never be truly independent. Therefore central banks should simply ignore their balance sheets and not treat low or negative net worth as a problem.” Are these statements true? If some of them are true, is there nonetheless a counterargument to the conclusion?

The central bank can indeed always pay its bills by printing money, so long as money retains some positive value. In that sense it can't become bankrupt. Also, it is true that if a central bank is concerned about its balance sheet, then it is possible for it to get into a situation of negative net worth, which can be corrected only by an action by the fiscal authority (e.g. issuance of interest-bearing debt that is handed over to the central bank). It might be argued that if the CB has to make such a request of the fiscal authority, it can't be truly independent. It is likely that in the face of such a request, the treasury or legislature will raise questions about the monetary policy that led to the request. The third statement, though, which follows the “therefore”, does not actually follow from the first two. A CB with negative net worth can pay its bills for goods and services, but it will face bounds on its ability to control the price level. In effect, being forced to use seignorage to pay bills and thereby increase the inflation rate is a sort of bankruptcy, at least if the public has regarded the bank as committed to maintain the value of currency and nominal government debt. So a CB that is committed to controlling inflation or the price level does need to be concerned about its balance sheet, and it ought to try to insure that the fiscal authority understands its policies and is ready to provide fiscal backing if the policies lead to balance sheet problems.

- (5) (30 pt) Maćkowiak and Wiederholt are attempting to explain some facts that are apparently inconsistent with menu-cost explanations of nominal rigidity. What are those facts, and how do Maćkowiak and Wiederholt explain them? In what way or ways do they deviate from consistently treating their economic agents as optimizing subject to a Shannon capacity constraint?

Maćkowiak and Wiederholt observe that price changes at the micro level are frequent and large, yet aggregate price indices respond sluggishly to macroeconomic disturbances. This does not fit well with a pure menu cost model, since the frequent large price changes imply low menu costs, which in turn should imply rapid response to all kinds of disturbances, including macro disturbances. They argue that a rational inattention model can account for these facts, since it implies that if micro-shocks are large and frequent compared to macro shocks, price-setters may pay more attention to micro shocks and thus respond to them more quickly. Their formal model, though, does not treat price setters as able to “code” information arbitrarily, in contrast to the Shannon theory. Though there is a single capacity constraint, the micro and macro sources of uncertainty are constrained to be independent conditional on agent actions. Optimizing agents would choose to make the micro and macro information ex-post dependent, even if ex ante they were independent. It is likely that the MW conclusions would hold even if they relaxed this constraint.