EXERCISE DUE MONDAY, MARCH 10

We will make accuracy checks on solutions of the (almost) linear-quadratic permanent income model. The model assumes an agent that maximizes

$$E\left[\sum_{t=0}^{\infty}(C_t - \tfrac{1}{2}C_t^2)\beta^t\right]$$

subject to, for each $t = 1, \ldots, \infty$,

$$W_t = R(W_{t-1} - C_{t-1}) + Y_t C_t \leq W_t \,. \tag{$*$}$$

We assume that Y_t is an exogenously given stochastic process, with Y_t i.i.d. and distributed uniformly over the interval $[0, .1]$ for every t. We also assume $R = \beta^{-1} = 1.05$.

It is conventional to study this model with $(*)$ replaced by

$$\beta^t W_t \xrightarrow[t\to\infty]{} 0 \,. \tag{$\dagger$}$$

This constraint (because it rules out W exploding upward) makes the transversality condition unnecessary, and it leads to a linear decision rule, with C_t a linear function of W_t. Also, it is easily shown that it leads to both C and W following random walks.

With the non-negativity constraint $(*)$, however, we instead get a highly nonlinear optimal decision rule. Consumption that exceeds the satiation level ($C = 1$) is obviously a mistake in this case. It occurs in the conventional solution because the constraint $(\dagger)$ requires dissaving if wealth climbs too high.

The exercise is to compute both Marcet-den Haan and "Judd" style accuracy checks for three possible decision rules for this model. The idea is to see whether they give clear indications of the inaccuracy of the solution and to extract any available insight about which direction to go in improving the solution. The three decision rules you are to check are:

$$C_t = \min\left\{\beta(\bar{Y} + (1-\beta)W_t), W_t\right\} \tag{1}$$

$$C_t = \min\left\{\beta(\bar{Y} + (1-\beta)W_t), W_t, 1\right\} \tag{2}$$

$$C_t = \text{ your guess of a function of } W_t \text{ that improves on the two above.} \tag{3}$$

Note that the first two of these are, respectively, the solution to the problem with $(\dagger)$ replacing $(*)$, modified to make $(*)$ nonetheless satisfied, and a solution that further modifies the conventional solution so that it never implies borrowing. The first will generate stationary stochastic simulated paths in the limit as the the length of the simulated path increases. The second will generate eventual convergence to a constant $C_t = 1$ (though, unlike the optimal solution, it will not necessarily keep C at 1 forever after the first time it reaches 1).

In generating simulated paths to apply the Marcet-den Haan approach, it is probably best to restart many times from, say, $W_{-1} = 0$ as an initial condition, rather than running a long simulation. Why?

It is part of the problem for you to decide which W_{t-1} values to look at in applying the Judd approach.